

**The development of children's theories:
the case of horizontality**

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To Fabio

Declaration

I declare that this thesis has been composed by myself and that the research reported therein has been conducted by myself unless otherwise stated.

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Edinburgh, 29th September 1990

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Abstract

The thesis sets out a domain-specific framework to deal with the problems of early competence and heterogeneity in conceptual development. According to this theoretical framework, cognitive development does not involve general change in the child's representational capacities, but change in local structures of knowledge, or theories. Within domains, knowledge acquisition proceeds in a stage-like fashion and can be characterised as the process of extending and restructuring theories. This theoretical approach is applied to the development of the concept of horizontality.

Three empirical studies investigate children's ability to orient and to reproduce the horizontal orientation of different planes in equilibrium: a table-top, a cross-bar and the water-level. For each context, a *décalage* between the correct orientation of the plane when it is perpendicular to its support and the correct orientation when it is at some angle, is observed. This finding is interpreted as evidence of the existence of two systems of spatial reference: object centered and framework centered. It is suggested that children's difficulty in reproducing the correct orientation of horizontal planes such as the water-level, arises from the conflict between the two systems of reference. The transformation of children's understanding of horizontality is thus explained, not as the acquisition of a new system of reference, but as the increasing ability to make the correct choice of reference axes and to resolve the conflict between the different axes assigned to the display. Knowledge of the dynamic and physical properties of objects is shown to play a major role in the development of this competence.

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Chapter 1: Introduction

1.1 The general problem

Although many years have passed since Piaget's first studies on the general categories of knowledge: space, time, causality, number, etc., his work still remains one of the fundamental references for the study of cognitive development. In the domain of spatial cognition, for instance, Piaget's developmental hierarchy of geometrical systems in topological, projective and euclidian, is still considered a useful general framework to which contrast new empirical evidence. A very recent article by Miller and Baillargeon (1990) is a case in point; it explores the notion of length and distance and attempts to challenge Piaget's hypothesis of the developmental primacy of a topological representation of space.

It is true however, that while Piaget's characterisation of the nature of the development of specific concepts is still taken as reference or as a "straw man", his general theory of cognition has become much less influential. The reasons for this decline have principally to do with the lack of clear evidence in support of Piaget's theory of general stages of development. On the contrary, developmental psychology over the last 20 years, has substantially shown the heterogeneity in children's conceptual growth. There is in fact little simultaneity in the acquisition of concepts that should share the same operational structure and the same logical complexity, according to Piaget's theory. Furthermore, there is a general consensus, that content and context play a major role on children's performance. Not only different versions of the same tasks can be solved at different ages, but by simplifying adequately certain task conditions, very young children can exhibit competence that Piaget had claimed was reached late in the operational stage. Studies on infants have also provided evidence of extremely precocious competencies that have led to attribute the young child with a wide range of innately specified primitive structures.

These findings raise the issue of representing the change in children's competence in development. Recent proposals suggest that conceptual change should be characterised at the level of domains and not at the level of the general representational capacity of the child. Children's conceptual growth is seen not as a qualitative transformation of their instruments of knowledge i.e. the representational format and their inferential apparatus, but as a transformation of the content of their domains of knowledge. The fact that from this perspective, cognitive evolution is characterised as knowledge acquisition does not preclude the possibility that there is a stagelike process

of restructuring of their knowledge structures, simply the claim is that restructuring intervenes at the level of local structures. Stages are therefore defined within and not across domains as Piaget had proposed.

The processes that are being invoked in the domain specific perspective, to account for the change in the child's competence and understanding as he evolves, share some of the features of another field which occupies itself with knowledge progress, namely History of Science. Current approaches are proposing that conceptual change is the outcome of processes such as generalisation, extension, conceptual specification, coalescence and differentiation or the shift from implicit to explicit knowledge. In particular researchers such as S.Carey or A.Karmiloff-Smith are drawing a parallel between the child and the "theoretician". Local structures of knowledge are defined as theories which as scientific theories, are coherent systems of explanation. At any stage of development therefore, regardless how limited his competence in a domain may appear, the child is attributed a coherent system of understanding. Developmental progress is therefore explained by the restructuring of the child's theories that with development acquire greater generality and explanatory power.

This perspective opens a new set of empirical questions. If in the last 20 years the preoccupation seemed to be that of identifying the moment in which the child acquired the mastery of a certain notion, be it classification, number conservation or spatial reference, the focus now would seem that of capturing the *transformation* of children's concepts in development. The central question is that of capturing the coherence of children's conceptions at different stages and accounting for the extension of children's theories from one stage to the next. This means providing an adequate interpretation of the content of the child's theory and the extent of his understanding at different moments of development. I suggest that this can be achieved by examining the child's ability to cope with a series of situations probing a specific concept, or in other words a battery of tasks analysing experimentally the competence levels underlying the steps of elaboration of the concept. Evidence from replications of classical Piagetian tasks has amply demonstrated how different versions of a task could elicit anticipated success. The modifications introduced by these studies were generally analysed in isolation by comparing the results at Piaget's task to results at a simplified version. What I envisage on the contrary, is a *systematic* comparison of multiple variations of a task. Therefore the objective of this type of strategy is to describe and explain orders and sequences in the solution of related tasks; sequences that mirror the progressive extension of the knowledge domain investigated.

1.2 The empirical problem

In this thesis I examine the general questions raised above with respect to a specific developmental phenomenon: the acquisition of horizontality. In 1947 Piaget & Inhelder reported an interesting developmental phenomenon; children younger than 8 or 9 do not represent the surface line of a liquid always horizontally. In the well known water-level task, children were presented with a bottle half full of water, the bottle was then oriented at different angles and children were asked to draw how the water-level was oriented for each tilt of the bottle. Typical errors consist in representing the water-level as invariably parallel to the bottom of the bottle, as a line joining the angles of the bottle or as a curved line partially parallel to the sides of the container and partially directed towards the beaker of the bottle.

Piaget & Inhelder designed the water level task in order to probe the children's representation of the system of vertical and horizontal coordinates. Subject's errors in representing the orientation of the water-line were interpreted as evidence of the lack of a stable system of spatial reference. In other words, the authors explained children's failure in the water-level task as the incapacity to relate the position of the water, the position of the container and the environment fault of a fixed reference system to which coordinate all of these elements. The elaboration of an invariant system of reference like the horizontal and vertical axes, is a crucial acquisition in the development of the spatial organisation of the child as it allows to relate all positions and displacements within a unified grid of relations.

Piaget inserted these results within the perspective of his theory of the development of spatial cognition. There is a first stage in the development of spatial representation which is attained towards the age of 5, in which the only spatial relations grasped by the young child are topological relations like inclusion, proximity, closure etc.. At this stage children have no understanding of euclidian relations as parallelism or perpendicularity, and are incapable of fixing an invariant immobile reference frame to which relate the positions and displacements of moving objects. The acquisition of topological geometry is achieved at the pre-operational level as it does not, according to Piaget, require the representational mobility and coordination of operational groups. Only at the operational stage will the child represent euclidian features of space and thus be able to represent space as an "empty container" organised around an invariant coordinate system of reference.

Subsequent research on the water-level task has shown not only that certain modifications of the task allow younger subjects to successfully represent the horizontal orientation of a plane, but that a considerable proportion of adults fail the

Piagetian task. These findings cast some doubts on the hypothesis that children have a radically different spatial representations from adults and that they lack the competence to represent horizontal and vertical coordinates. In other words the characterisation of the development of the coordinate system suffers some of the typical problems of Piaget's account of the preoperational stage, lack of detail in the analysis of the intermediate phases, abstractness of the concept that is seen as the output of the developmental process, impossibility of integrating data showing partial or context bound success to tasks sharing similar structural constraints to the original task.

It remains to be explained however, why children systematically fail the water-level task, and under what circumstances they have the capacity to reproduce the horizontal orientation of a plane. The thesis will provide new evidence on children's ability to represent the orientation of objects and planes and will attempt to characterise the organisation of the underlying competence. In particular the representation of the direction of gravity as a referent for horizontality, will be shown to play a major role in correctly reproducing the orientations of planes. The transformation of children's understanding of problems of orientation will be interpreted, not as a sudden acquisition of a new system of reference, but as the progressive discovery of the relevance of gravity as a referent for predicting and representing the positions that objects assume in the world.

1.3 Plan of the thesis

In Chapter 2, I give a brief presentation of Piaget's theory of general stages of cognitive development. I then introduce some of the arguments that have been brought against Piaget's hypothesis of overall logical structures. The evidence of precocious successes in operational tasks, of low correlations in performance across different conceptual domains, of contextual biases in adults' logical reasoning are some of the factors that have led to the alternative proposal that development does not occur as the gradual transformation of the child's overall representational capacity, but as the transformation of specific domains of knowledge. According to this view, conceptual change proceeds at the level of local structures which develop with some independence, at different rates.

Theories of cognitive development that invoke domain specificity are presented, with particular attention to those that have introduced the metaphor of the child as a "theory builder": the progress of the child's domain-specific knowledge, like the progress of scientific knowledge, goes in the direction of greater explanatory and predictive power.

I argue that by invoking the notion of "theory" to characterise local conceptual structures, our account of development inherits some of the desirable properties of the Piagetian system, namely that development proceeds in stages through the progressive restructuring and extension of existing conceptual structures. At the same time, the domain-specific account has the advantage of invoking local restructuring of theories of the domain, as opposed to the global logical restructuring of Piaget. I will conclude by suggesting that this alternative approach can allow us to raise new questions about development and to provide a unified account of a variety of data on conceptual development.

In Chapter 3, I present some methodological considerations about the analysis of domain-specific development. A domain-specific perspective raises in fact new empirical questions that require a specific methodological treatment. I argue that the basic developmental question is redirected from whether the child *has* a (operational) concept or not, to that of *charting the transformation* of the concept through a sequence of stages. Analysing the concept's transformation as opposed to identifying when a concept is acquired by the child, determines a shift from a method based on critical tasks to a method based on batteries of tasks. While critical tasks discriminate between subjects who have acquired a certain competence from subjects who have not, a battery of tasks, all probing the same concept, discriminates the conditions in which a subject is able to deploy his knowledge from the conditions in which he is not, or in other word the boundaries of a concept at a stage. From the orders in which the set of tasks is solved, levels of problem-solving ability emerge from which the underlying conceptual organizations can be inferred. Interpretations of the underlying conceptual structures (or theories) are then advanced and tested by devising new tasks which vary the context of application. A task can be designed to be compatible and thus accessible to the subjects who have a particular theory, in which case it is predicted that the task will elicit correct responses. Failure to confirm this hypothesis would lead to reconsider the interpretation given to the content of the child's theory.

I propose a method of hierarchical analysis to identify the order in which the battery of tasks, dealing with the domain, are solved. The advantages of the hierarchical analysis are that it makes possible a) to integrate the data of early competencies, b) to evaluate the hypotheses that, within a domain, the child develops through a sequence of ordered stages and c) subsequently to test precise hypothesis about the competence underlying the stages. A specific statistical method for hierarchical analysis will be presented in detail: Hildebrand, Laing & Rosenthal's prediction analysis of cross-classification.

In Chapter 4, I examine in some detail the existing literature on the water-level problem. First, I give a detailed presentation of Piaget & Inhelder's original study, of the stages of development they describe and of the account they provide. This explanation is then related to the wider context of their studies of spatial cognition. Second, I review the replication studies which provide the basis for a discussion of the cognitive processes invoked to explain the acquisition of the system of coordinates. The data from the replication literature, in fact, provides some evidence that modified versions of the task elicit successful responses before the Piagetian version of the water-level problem. In particular, changing the physical context of the orientation task by eliminating the bottle and the liquid makes subjects orient the plane horizontally before they can do so in the case of the liquid surface. Furthermore the finding that a considerable proportion of adults fail the water-level problem raises some questions on Piaget's interpretation, namely that subjects who fail the task lack the representational mobility and operativity to coordinate multiple spatial relations into an organised system.

Partial success in the water-level task, success at some modified versions of the task, etc. indicate that the child can make use of some of the relations of euclidian geometry which Piaget is willing to attribute only at the last stage of the development of the coordinate system. At the same time, however, the concurrent failure in some of the conditions of the water-level problem suggests that this prior level of spatial representation lacks some features of adult's spatial organisation. Neither Piaget nor any of the following studies provide a specific characterisation of this intermediate level. In the conclusions, I suggest that it is necessary to capture the coherence underlying the responses of subjects in this period of development in order to provide a complete account of the genesis of horizontality.

In Chapter 5, I introduce the first of three experiments designed to study the nature of children's representation of the horizontal/vertical coordinate system in the intermediate stages. This experiment replicates the water-level task but analyses more closely two factors that seem to affect the performance in the task: the angle of tilt and the shape of the container. The literature review suggested, in fact, that there is no abrupt transition from failing all condition of the Piagetian water-level task to succeeding them all.

The experiment is designed to contrast the two conditions, round and square, and two set of tilts of the container, orthogonal and diagonal. On the basis of these categories, orders are established between the solution of the task in the different conditions; the

orders allow us to make hypotheses as to the extension of children's competence prior to their success in all the task conditions.

The results indicate that children first succeed when the container is in one of the orthogonal positions (tilted at 0° , 90° , 180°) and when the container has a rounded shape. I advance the hypothesis that success in these conditions of the water-level task and the concurrent failure in the diagonal conditions may indicate a level of spatial organisation centered on within-object relations. Children may be able to relate the water-level with the elements of the container and determine the horizontality of the liquid surface only within the restricted space of the bottle and not with respect to an external general frame base on the direction of gravity and its perpendicular.

In chapter 6, I present the second experiment which investigates the role of the physical context in the orientation task. Alongside the water-level problem, children are asked to represent the orientation of a crossbar in a balance. While preserving the spatial structure of the Piagetian task of horizontality, the new balance task does not involve the complex dynamic properties of liquids. The experiment is aimed at testing the hypothesis that the child goes through a stage of object centered spatial relations which underlies the anticipated correct solution to the orthogonal orientation of the bottle as well as of the new balance task.

The results confirm that orthogonal effect, i.e. when there is a coincidence of the axes of the environment with the internal axes of the object the horizontal orientation of the plane is more easily reproduced. This is found to be the case with the balance as well as with the water-level task. Furthermore an anticipation is found in the solution of the balance task with respect to the square container, on the contrary the balance and the round container are solved concurrently.

Chapter 7 presents the third experiment of the series. The experiment focuses on young children aged between 3 and 6 years, an age range about which there is very little data. I replicate the water-level and balance task in order to assess the order of acquisition of the two conditions and introduce a third situation which is especially designed for young children. A practical task of orientation of a table top to maintain a ball in equilibrium is presented alongside the two previous situations. As in the two other tasks, the table task preserved an equivalent spatial structure, and two conditions are contrasted: one in which the object is in a orthogonal position and one in which the object is in a diagonal position. The objective of the study is to identify precocious competences in orienting horizontal planes, and to further test the generality of the hypothesis of an "object centered" stage of spatial representation.

The "orthogonal effect" is found once again in the table task , the diagonal condition however is solved well before the other two tasks showing a very early competence to orient a plane horizontally. The general hierarchy that emerges from this final study indicates that there is no "once and for all" transition from an object-centered to a framework-centered spatial system.

Chapter 8 is the concluding chapter of the dissertation. I discuss the import of the results of the experimental work on Piaget's explanation of the development of the horizontal and vertical coordinate system. I argue that the results challenge the view that there is a radical shift from a stage in which children do not possess a euclidian spatial framework to a stage in which children have acquired a coordinate system of reference. I propose that children come to progressively discover the relevance of the direction of gravity as a referent for the position objects assume in the world depending on the understanding of the dynamic properties of the objects involved.

Chapter 2: A domain specific approach to cognitive development

2.1 Introduction

In 1983 Gelman and Baillargeon concluded their review of Piagetian concepts by claiming that the developmental data gathered in the sixties and seventies did not support the idea of major stages in cognitive development proposed by Piaget.

Over and over again, the evidence is that the preoperational child has more competence than expected. Further, the evidence is that the concrete-operational child works out concepts in separate domains without using the kind of integrative structures that would be required by a general stage theory. In addition there is evidence in some cases that the structure underlying the way preschoolers reason about a problem is much like that used by older children and even adults..... (Gelman & Baillargeon 1983 pp. 214)

There are many reasons for the decline of Piaget's theory of general structures which have to do both with the coherence of the theoretical apparatus and problems emerged in the empirical validation. There is in particular one class of data that was accumulated in the sixties and seventies by developmental psychologists attempting to replicate Piaget's studies, that in my opinion has been particularly prejudicial to Piaget's interpretations: the evidence of early acquisition of operational notions. It is the case that for the majority of tasks Piaget used to investigate the operational period, a slight modification of the experimental situation (be it interactional, contextual or perceptual) can elicit successful responses in subjects that fail the classical Piagetian task. In most of these studies the structural constraints of the original task have been maintained and particular care was taken not to introduce changes that transformed the logical requirements of the task. Thus the anticipated solution to these tasks has been interpreted as evidence of early competence, i.e. under certain circumstances young children can operate with concepts that are formally equivalent to those of older children. Well known researches of this type are McGarrigle & Donaldson's modified conservation tasks (1975), Markman's modified class inclusion task (1978), or Bryant and Trabasso's modified seriation tasks (1971) just to pick a few examples. The difficulty of integrating such evidence in the explanatory framework of Piaget's theory was amply discussed at that time and can be resumed as the problem of procedural *décalage*; according to Piaget's model performance factors should not allow a child of a certain stage to solve problems of an operational complexity superior to his level.

The multitude of replication studies of Piaget's tasks, each one set up for its own specific objective, have conjointly contributed in creating the current zeitgeist in developmental psychology. There is in fact a general consensus in believing that preoperational children have far more competence in a variety of conceptual domains than what was thought in Piaget's time. In fact, the clear cut distinction between pre-operational and operational children Piaget had introduced, has appeared grossly inadequate to capture the nuances of diversities in children's conceptions at different moments of their development. Among other things, the replication literature has exposed the fundamental role of content on the child's reasoning. It has done so not in the classical Piagetian tradition of considering the child bound to contextual or perceptual factors, but on the contrary by suggesting that given the right content (e.g. a content they are familiar with), young children do not differ qualitatively in their reasoning strategies from adults or older children. Miller and Baillargeon (1990) for instance, provide an up to date example of this type of investigation for the domain of spatial cognition. In their study, they reach the conclusion that children cannot be described as using a radically different geometry than adults.

Apart from undermining Piaget's model of stages of logical organisation, these results have contributed to reopen the important issue of the locus of developmental change, and in particular whether cognitive development entails a transformation of the child's representational format. Again and again the following question is being asked: if children do not differ from adults in their representational capacity, logical competence or computational power (however one wants to call it), how is one to account for the difficulties they encounter in a range of tasks solved instantly by adults, and how is one to explain the nature of the cognitive change they will undergo during development?

A proposal that has emerged in recent years denies that there are general across the board qualitative differences in the child's and adult cognition and considers that cognition is organised in separate domains which develop independently at different speeds. Within a domain specific hypothesis the explanation of developmental change is transferred from general principles of conceptual organisation to local conceptual structures. Among the few attempts to describe of what nature these domain specific structures could be, the most clear proposal of recent years, has been to characterise the stages of the development of a concept in terms of theories (Carey 1985, Karmiloff Smith 1988, Gopnick 1988). Within this perspective the process of domain specific knowledge acquisition is seen as inheriting some of the properties of scientific theories and their development in history. The notion of theory has structural characteristics that allow to express the radical discontinuity from a stage to the next. It

also involves a property of locality which is essential for a domain specific description while presupposing an internal organisation of a holistic nature. In this sense by saying that the child constructs increasingly more articulated and adequate theories of domains, the desirable Piagetian notion of restructuring and discontinuity can be maintained, while at the same time considering that change occurs at the level of local systems of conceptual organisation rather than at the level of general domain independent structures.

The chapter is organised in three sections. In the first I briefly introduce some elements of Piaget's theory and in particular the notion of general stages of development. In the second I recall some of the arguments and data that were brought against Piaget's hypothesis of general logical structures. In the third part I introduce the domain specific perspective with a particular focus on the approaches of Carey and Karmiloff-Smith.

2.2 A theory of the development of general structures

If one was to attempt to characterise Piaget's genetic epistemology in three key words, these would be constructivism, interactionism and structuralism. Knowledge is the outcome of the interaction between an acting organism and a changing environment. The individual is endowed with elementary cognitive structures to which experiences are assimilated and which in turn accommodate to the external pressure of the environment. In the course of development, these internal cognitive structures are complexified and become more equilibrated to produce progressively more adequate and objective knowledge of the world. These adapted structures thus are not given prior to experience, but derive from the individual's action and are constructed in interaction with the world. Thus the tenets of his epistemology are articulated: construction of logical structures through interaction with the environment.

2.2.1 Scientific knowledge and the growth of logic in Piaget.

Piaget's biological background shaped his outlook on cognition, and determined his definition of what a theory of knowledge should be. A theory of knowledge for Piaget, is essentially a theory of the adaptation of thought to reality, by the progressive increase in rationality of the individual. The development of cognition is seen, therefore, as a process of growing adaptation. The increase in the adequacy of knowledge corresponds to an increasing control of the organism on his environment.

The function of cognition is hence to augment and optimise the organism's exchanges with the world. Different levels of development are characterised by different levels of interaction and understanding of the environment. The child is seen as developing more and more adequate (in the sense of coherent and successful) means to represent the world and thereby increase his capacity to anticipate, explain and resolve events. The growth of knowledge is therefore, the process by which a subject constructs his interpretation of the world and thereby adapts to it.

The cognitive system, according to Piaget, is governed by an independent regulative mechanism set to augment its knowledge of the world i.e. reflective abstraction. The process is driven by the research of coherence, control and success and more generally, greater equilibrium. During ontogenesis this regulation arrives to a progressive optimisation of the cognitive activity and the greater stability and mobility achieved, allows the organism to solve and anticipate more and more problems.

This increasing equilibrium and operativity, relies on the progressive construction of a more powerful system of logical relations. For Piaget in fact, cognitive adaptation had to correspond to an augmentation of reasoning power. The level of complexity and adequacy of the subject's understanding is dependent on the level of representational power of his cognitive organisation as a whole. The autoregulative system relying on the mechanism of reflective abstraction constructs progressively more powerful "structures of the whole" defined as logical systems. For Piaget there is no doubt that the process of equilibration tends to greater and greater rationality which relies on the formation of certain kinds of structures which define permissible transformations and relations. As Kitchener points out:

In fact one can see his (Piaget's) entire theory of cognitive development as an attempt to construct a theory of historical rationality rooted in biology. (Kitchener 1986, p. 191)

The strategy Piaget designed to analyse the process of knowledge growth, in the individual, proceeded on two fronts: the first task was to explain the formation of scientific knowledge, and hence address the question of how the growth of particular domains of knowledge corresponded to an increasing adequacy of these concepts. The domains of knowledge he explored ranged from space, to time, to number etc. The second, was to identify the common elements in the individual's development and hence the general mechanisms underlying the construction of all domains of knowledge. In this sense the identification of general organization laws of the cognitive system was achieved through the analysis of the content of children's intuitive theories of the world or knowledge structures. Although the study of the cognitive system

could only be achieved by examining the categories of knowledge and norms that individuals construct in development, ultimately, Piaget considered that cognitive development is governed by *universal invariant laws of reasoning* which determined the level of adequacy of children's representations.

Hence, underlying Piaget's description of knowledge acquisition, was a theory of cognitive development as a progressive construction of general logical structures or "structures d'ensemble", each stronger and including the preceding one. Each structure represents a kind of logic, a set of formal or semi-formal relationships. There is first a logic of action involving a set of relationships, correspondences, orders and classifications, elaborated at the end of the sensori-motor stage around 18 months of age. Then it is followed by concrete operational logic involving groupings of classes and relations, elaborated in the concrete operational stage around the age of 8 or 9. Finally formal operational logic with its specific structure the INRC group is constructed during adolescence and culminates in the stage of formal operations. Development proceeds therefore, through four stages each characterised by an increasing level of logical operativity: the sensory-motor stage, the pre-concrete operational stage, the stage of concrete operations, and the formal stage. Piaget claims:

Each logical structure is characterised by an instrumental usage within a stage.
(Piaget 1975, p.6).

Within a stage, operations across knowledge domains derive from the same general operational organisation and are hence supposed to be synchronous. The same process of reorganisation is evident in the order of acquisition of different notions, which reflects the reoccurrence of the process in identical fashion across all knowledge domains. For example, the grouping structure with its property of reversibility, elaborated at the stage of concrete operations, is reflected in the reasoning patterns underlying the application of such notions as conservation, seriation and classification. The new operation of reversibility brings about radical transformations in the domains of number, space, causality etc.. General structures develop through general reorganisations which are content independent. The construction of each domain of knowledge is hence, the exemplification of the unfolding of the process towards necessity, i.e. the logical-mathematical structures defining mature scientific knowledge.

It must be stressed however, that in Piaget's theory, the relation between knowledge domains and logical structures is not just unidirectional, operations structuring knowledge content; if we look at the process of development in an individual, there is an important effect of content on structure. The question is how does the subject's use

of knowledge and his experience of the world, modify his structures. Piaget argues that it is incorrect to think that logical operations are simply "attributed" or applied to objects, without an interaction with the physical world:

The exchange between causal contents and operations consists in an action of the contents on the cognitive functions, which facilitate the construction of the logical form, which in its turn reacts on the contents." (Piaget & Garcia 1971, p.50).

The new operations are in turn tested by new experiences. Hence development of cognitive structures seem to follow a cyclic chaining of external observations and internal coordinations. The extensions of cognitive operations enables the subject to capture new properties of the environment. These new properties require new operations that again can serve to discover new properties. For every progress of the logical structures (by reflective abstraction) corresponds a refining of experience or, in Piaget's terms, a phase of empirical abstraction. This new level of understanding opens new problems and questions which requires new relational capacities.

The notion of structure which is linked to the notion of stage has, as we can see, a fundamental role in Piaget' theory. The hypothesis of discontinuity which underlies the idea of stage is an essential element of constructivism. In fact Piaget opposed to the behaviourist view of a linear accretion of knowledge through direct experience, the idea of structures as organised systems by which the individual assimilates his experiences of the world. By introducing the idea that children structure their knowledge in systems of increasing adequacy which develop through a process of restructuring, he claimed that there is not a linear accumulation of knowledge, but that conceptual development proceeds through discontinuous phases which are qualitatively rather than quantitatively different. The radical discontinuity that characterises the shift from one structure to the next, is embedded in the notion itself of structure as a holistic system of relations.

It is within this epistemological framework that Piaget's notion of stages is set. Let us now consider more closely the characterisation of stages that Piaget provides.

2.2.2 Piaget's theory of stages, what do they predict?

It is worth examining the criteria Piaget gave for a stage, as stages have not only a central status in the theory but have been the object of the closest scrutiny and experimental validation from the developmental community. We will state some of the

principles involved in Piaget's definition of a stage before analysing the arguments that were brought against them by post-Piagetian literature.

- a) the sequential order of succession of stages must be constant; a stage cannot be skipped, an order cannot be inverted and regression is impossible;
- b) the structural elements of a lower stage are an integral part of the structure of the following stage, although present in a new form and inserted in a new network of relations;
- c) each structure is a "structure of the whole", such that once a structure is achieved one can determine every operation it covers;
- d) each stage contains a process of preparation and a final level of achievement;
- e) each stage represents a certain degree of equilibrium and subsequent stages are more complex and more equilibrated;

The most important psychological predictions that the properties of stages entail or have been taken to entail by the developmental community are three:

- 1) Since elements of stage constitute a structure of the whole and cognitive elements of that structure are interconnected logically with other cognitive elements, once a subject has mastered a task and showed the acquisition of the underlying competence, there should be synchronous acquisition of the other logically equivalent related concepts. This is also known as the issue of horizontality of structures of cognitive development, or to the related notion of homogeneity of cognitive organisation.
- 2) There is radical discontinuity between stages. Development from one general stage to the next corresponds to a qualitative shift and the emergence of a new system of thought.
- 3) Variations in the context or content of the task should not radically affect the performance of subjects in a task, in particular when the operational requirements of a task or problem are kept constant. Only minor anticipation or retardation in the solution of the task should occur as a consequence of changes in environmental variables.

There is a wide range of opinions, at least as many as there are developmental psychologists, as to the status of such predictions in Piaget's theory and as to the importance of confirming such predictions in order to validate or falsify Piaget's theory. It is a debate far too vast for us to enter here, we will cavalierly assume that there is a degree of agreement and that the data collected in the past 25 years has in fact been able to assess some of these predictions.

It is important to note that these predictions are derived from Piaget's theory and are not formulated explicitly by himself. Piaget's concern with the epistemic subject was greater than with the psychological subject, therefore the actual prediction of individual behaviour was peripheral to his interests. The predictions of homogeneity across stages, its empirical assessment and critique corresponds more to the preoccupations of psychologists which have attempted to make a working psychological model of Piaget's theory. This enterprise of systematising and testing the Piagetian framework has been an essential step in the exploration of the theory it has not however always taken into account the different set of objectives that characterised genetic epistemology.

Two of these predictions in particular have raised the greatest bulk of research and have been found most problematic: *the homogeneity of performance across domains, and the invariance in the order of acquisition of notions of different operational level.*

2.2.3 Homogeneity and invariant sequences

The theory of general stages was seen as claiming that tasks of the same operational level be solved synchronously. Although much contention appeared in the literature as to the real importance of homogeneity and to how stringent synchronicity was to be taken (Flavell & Wohlwill 1969, Flavell 1971 e.g.), basically once taken into account the preparatory, intermediate and final phases of a stage structure, it was agreed that tasks of similar operatory complexity should be solved concurrently if the idea of general structure has to have any meaning (a position defended for instance by Brown & Desforges 1979). As Karmiloff-Smith (1984) suggests:

Although the structures underlying some behaviour may be symptomatic of the immediately preceding stage, one would not expect a stage account to allow for a child to be at, say, stage 3 in a certain number of problems and stage 1 in others. Rather, the child is predominantly, at, say, stage 3 in most problems and in some, perhaps at stage 2. (Karmiloff-Smith 1984, p.41)

1) A considerable number of studies tested the synchronicity of development in children of the wide set of concepts described by Piaget, new measuring techniques were employed, simplified and complexified versions of the tasks were examined. The list of these researches is practically inexhaustible (an extended overview is offered by Brown & Desforges (1979), and Gelman & Baillargeon 1983). The absence of correlations between tasks reported also by researchers sympathetic with Piaget's positions, like Dodwell (1962), Laurendeau & Pinard (1968), Tuddenham (1971), Pascual-Leone (1976), and more recently Lautray, Rieben & Ribaupierre (1985), have implied a serious reconsideration of the general stage prediction. Low degrees of association were found between formally equivalent problems involving number or space (Lovell, Healey & Rowland 1962, Dodwell 1962), both within and across domains. The data suggested that the assignment of a subject to a given stage is dependent upon the task used as criterion. Subjects were in fact found to be in one stage for one task or set of tasks and in another stage for another set of tasks. Evidence of heterogeneity seemed the rule rather than the exception as Piaget suggested.

The data on heterogeneity may be, however, considered insufficient to knock down Piaget's hypotheses of a general structure on which the full range of abilities of a developmental stage are based. It could be argued in fact, that all the subjects examined in the correlation studies could have been in an intermediate phase; or that it is the characteristic of concrete operations to be applied to different contexts at different moments. However, when coupled with the evidence of early acquisitions, the argument holds more ground.

2) Within Piaget's account of single knowledge domains, there is invariance in the order of acquisition of notions of different operational level. In the development of a concept, a sequence must be respected which corresponds to the hierarchical model of levels of operations.

If one interprets Piagetian theory to mean that performance within each domain is based on operations that are organised into a well integrated, reversible structure, then one might expect to find high correlations between tasks testing abilities assumed to be derived from that same structure. (Gelman & Baillargeon 1983, p.170)

The hierarchy of tasks described by Piaget for particular concepts was not always replicated, as in the well known study by Kofsky (1966) on classification. On the basis of Piaget's analysis of the development of classification, Kofsky identified a ranking of the 12 tasks used by Piaget. However only 30% of the subjects fit the pattern expected. For instance, it was found that the ability to multiply classes in a

cross-class matrix, does not predict the ability to solve class inclusion as Piaget's theory would expect. Evidence of this kind makes it problematic to support the Piagetian argument that within a domain, some concepts were fundamentally dependent on the acquisition of a common structure evolving in an orderly sequence.

3) Finally, although there is space for some variation, in the Piagetian theory, the acquisition of a concept is fundamentally an all or none phenomenon, variations in the context or the mode of presentation of the task, should not effect the subject's understanding of the problem as long as the task's operational complexity is kept invariant. Hence once a child is developed to a certain stage, contextual variations should not regress him to a lower stage. Conversely task variations should not enable a child of a lower stage to understand problems of a higher logical complexity. Therefore, evidence of early success in a modified task, preceding the operative level that the task would require according to Piaget's theory, and evidence of adults not attaining formal operativity in the solution of certain concepts, has cast doubts on the model of a fixed sequence of stages in operational thought.

Early competencies have been exposed for most Piagetian concepts when modifications of the original tasks have led to systematic anticipated success. Since the modified tasks were designed with the aim of maintaining the logical structure of the task, while changing the pragmatic, linguistic or interactional context, the anticipated solutions suggested that children possessed the concept under scrutiny before Piaget had claimed. Failure in the original task was attributed to performance requirements which masked the real competence of the child. Paradigmatic examples of such studies are for instance the replications of the number conservation task carried out by Greco (1962), Elkind (1967), McGarrigle & Donaldson (1975). In all these studies modified conservation tasks elicited success among children who at the same time failed Piaget's classical task.

To take an example relative to the experimental situation that will be the focus of my empirical research, Mackay, Brazendale & Wilson (1972) replicated Piaget's water-level task (in which the child is asked to draw the orientation of the water level in a container half filled with coloured water), modifying the shape of the container simply by replacing the bottle with a side view of a puddle, and found significantly more successful responses than with the classical Piagetian apparatus. Younger children were found to reproduce the water-line horizontally in the modified task while still failing the Piagetian version. (I will analyse this effect and other related studies in detail in Chapter 4.)

Although in theory, as we pointed out previously, Piaget strongly defended an interactionist view where accomodation to the object experienced was as important as

assimilation to the subject's logical structures, the retroactive effects of experience is never analysed in great detail. What properties of the object and of the task, actually participate in modifying the structure is left unclear. This vagueness, unfortunately, seriously undermines the explanatory power of the theory. Experimental variables, such as those which produce the favourable conditions in the Mackay et al. task, have little or no status in the description of behaviour. The analysis of the process of acquisition of a task solution, is accounted for by Piaget, more in terms of internal coordination of schemes than by the interplay of environmental variables and the child's structures. Therefore the sort of data presented by McGarrigle & Donaldson on number conservation or by Mackay, Brazendale & Wilson on the horizontal can hardly be explained or predicted by the Piagetian theory.

It has been suggested that the data on early competence in the concrete operational tasks goes to indicate that many of the concepts that Piaget considered as the outcome of a long construction, are in fact innate or available very early in development. It can be argued convincingly (as Gelman & Meck (1986) have done for cardinal number) that the basic structure of some concrete operational concepts is present from the earliest stages of development.

However, two points are crucial: 1) The modified or simplified versions of the tasks are themselves the outcome of a developmental process; it is always possible to find younger children that fail the simplified task. 2) There is a developmental process that leads from the simplified to the classical Piagetian task; subjects may initially succeed at the simplified version while failing the classical task, but eventually they succeed in both. The questions that replication studies raise are therefore the following:

- if the two tasks probe the same concept, why can a subject first apply it to one situation and only later to another?
- what does the child that only solves the simplified version not know, that the child that solves both tasks knows?
- does the child that solves neither tasks, know nothing or can we find yet simpler tasks that he can solve?

These questions become the more pressing if we consider that for each Piagetian task (and specifically for those of the concrete operational period) there is a plethora of replications and a considerable subset of these have elicited anticipated solutions with respect with Piaget's original tasks. The challenge offered by the replication data is that of giving a unified account of the child's behaviour at the different versions of the task. If one considers for instance the literature on the horizontality task, there have

been a number of modifications examined which ranged from the shape of the container to the mode of presentation or of response. Although a few of these variations in procedure have elicited anticipated response with regards to the classical Piagetian task, there has been no attempt to relate all of the data together.

2.2.4 What picture of the child can be drawn?

The image of the preoperational child that has emerged from these studies is far more complex than previously described by Piaget. Nursery children and primary school children have been found to solve modified versions of conservation, classification or seriation tasks far before expected. The development of most concepts has been shown to be more gradual than Piaget had described and the route to its acquisition more tortuous. When paired with infancy research, the study of early acquisitions of numerical, spatial, classificatory, or causal concepts, has slowly undermined the viability of the operational model proposed by Piaget. Studies on infancy such as Bower's (1982), Butterworth's (1977), Mounoud & Bower's (1974), or Mandler's (1983), have brought undisputable evidence of babies' competencies that make Piaget's account of the sensori-motor child definitely old-fashioned. Similarly the image of the "illogical" adult as suggested by adult reasoning research has put limits on the upper boundary of formal operativity. Studies by Braine (1981), Wason and Johnson-Laird (1972), Johnson-Laird (1983), among others, showed that the content of logical tasks plays a fundamental role on adolescents' and adults' logical performance. Adults have been seen as not attaining formal reasoning in contexts which were found too abstract. Overall, the picture that all this research has proposed beyond the super-baby, competent child and inconstant adult, is of a fragmented development where content and context play a major role in the emergence of subjects' competence.

There are a number of conclusions that have been drawn from the replication of Piagetian experiments and from the attempts to validate or falsify his framework:

One lesson of modern research in child psychology is that accounts of how development proceeds can no longer ignore the possibility that at least some of the structures that underlie our system of knowledge are innate. (Gelman & Baillargeon 1983, pp. 220)

A second major conclusion is that it is extremely difficult to characterise the difference between young children and adults or older children in terms of radical differences in

the form of reasoning and the representational capacity. It is in fact very difficult to demonstrate that a child has a consistent level of thinking, a *representational format*, across all domains of knowledge.

Finally it has also appeared that despite the precociousness of acquisition of many concepts, young children are far from older children in the generality and stability of their use of such notions. A lot of improvement occurs with age. The young child can cope with limited task situations and can only succeed in the complex versions of the task (as Piaget's generally are) much later in development. It remains therefore to be explained in what way the task requirements differ and of what nature is the improvement in competence which accounts for the differential abilities in young and older children.

2.3 An alternative view

2.3.1 A domain specific perspective

What sort of explanation can be advanced to account for the developmental phenomena we have briefly mentioned above? It has been proposed that a coherent and systematic account of all the anomalies in the Piagetian data can only be provided within a domain specific perspective. Carey (1985) suggests for instance, that much of the evidence that has been taken to support changes in domain-general structures actually reflect domain specific structural reorganisations.

In some of their work, Piaget and his colleagues were very careful to keep separate domain-specific change and operational change. For example each chapter in *The child's conception of physical quantities* (Piaget and Inhelder 1941) receives two theoretical summaries. First, the children's concepts of weight, volume and density, as well as their understanding of such phenomena as the dissolving of sugar in water and the popping of popcorn, are diagnosed relative to successive models of matter. The construction of an atomistic model of matter is domain-specific conceptual change. Second the children's emerging conceptions of matter are diagnosed relative to the achievement of concrete operations and formal operations. In the work on the concepts of weight and density cited above, the interpretations in terms of the child's developing theory of matter are sufficient to explain the phenomena in the book; no appeal to concrete or formal operations is necessary. (Carey 1985 pp.190-191)

What Susan Carey is suggesting is that the different conceptions of matter that the child can be considered to hold at different moments of his development and which underlie the behaviour he exhibits in the resolution of related tasks, are themselves the cognitive entities that undergo change. The change in such conceptions can be explained independently without appealing to a second level analysis of the processing

or logical capacity which in turn would underlie the child's different concepts. What in Piaget was a first step in the study of the growth of knowledge in the individual, the study of the formation of specific domains of knowledge, can now take an independent status and become the locus of explanation of cognitive development.

A number of theories have been proposed which refuse any idea of general stages as Keil's (1984) domain approach, Carey's (1988) theory of theories, Klahr and Wallace's (1976) information processing theory, Karmiloff-Smith's (1988b) phase theory, Turiel and Davidson's (1986) domains theory and Feldman's (1986) localist approach. All of these approaches have in different ways suggested that cognitive development is fundamentally domain specific. This claim presupposes that there are no general stages in cognitive development and that the rate of acquisition of concepts depends on the domain of knowledge. No across-the-board principles determine understanding in all fields. More to the point, change occurs not at the level of the instruments of knowledge or the representational format, but at the level of individual concepts. It is assumed that the *same representational or computational systems* characterise cognition at all ages, but that difference between age groups is to be explained by an increasing access to the same structures, to gradual differentiation or to adoption of new strategies.

Domain specific theories vary significantly in the definition of a domain, in the mechanism of development they invoke, in what they attribute to the child as innate capabilities. Most approaches however agree on the idea that there are stage sequences within more local units of cognition or knowledge systems. Homogeneity is to be found therefore, within domains while heterogeneity is expected between domains. While Gelman & Baillargeon conclude their review of Piagetian concepts by claiming that there is little evidence to support the idea of major stages in cognitive development, they suggest that stages could be found within domains.

None of the foregoing points eliminates the possibility of there being within domain stages of development.....There is one possible way of retrieving the stage argument for within-cognitive-domain developments. This involves representing a given level of competence in terms of hierarchies of related concepts and then characterising each stage in terms of the dominant tendencies at a given time. (1983 pp. 214-215)

However a major question remains: if one is to take the view that the child is endowed with the capacity to represent concepts and relations which enables him to draw a series of inferences and conclusions about events and states of affairs, how is the change in conceptual development to be accounted for? Of what nature are the qualitative shifts that underlie the transition from a stage to the next within a domain? To quote Gelman & Baillargeon again:

To say that young children's reasoning structures are rich is not to say that they are the same as the adult structures. Indeed, we noted the many conceptual domains where (despite the presence of early capacities) young children's reasoning structures are nowhere near those of older children, which in turn, seem impoverished compared to adult capacities. As far as we are concerned to say that there are rich cognitive structures to start is just the beginning. There remain the questions of what are those structures and how they determine the emergence of advanced structures, given an appropriate range of experience. (1983 p.219)

The greatest risk of a domain specific approach which puts the burden of the explanation of cognitive development on the enrichment of preexisting concepts and more generally that characterises cognitive growth in terms of content acquisition, is that of falling into a linear accumulation model. The child would simply learn more and more and extend his knowledge by accumulating new experience. If the reasoning and representational capacities remain unchanged and are available since early in development and moreover a greater attention is given to the context within which conceptual abilities appear, it could be argued that cognitive development is just reduced to learning. The evidence of stages of development within domains strongly suggests on the contrary, that there are local discontinuities that seem to be more appropriately characterised as the restructuring of some organised conceptual system.

In my opinion therefore, the most pressing question that any domain specific or localist theories of cognitive development must face is that of providing an account of conceptual construction that preserves some of the properties of a general stage theory, like holism and discontinuity at the level of more local systems of cognition. The crucial question remains: *if one abandons a characterisation of cognitive change in terms of general structures of logical (or any other) nature, does this imply that knowledge progresses in a cumulative way?* Does talking of the acquisition of new contents mean that there is simply an addition of new concepts? Or can one talk of qualitatively different organisations of knowledge within local systems? Is it possible to envisage restructuring at a specific level?

2.3.2 Discontinuity within domains

Susan Carey makes a strong point in defense of the possibility of envisaging discontinuous development and restructuring at a local level:

Current research on knowledge acquisition, makes it clear that the contrast between structural development and the accumulation of knowledge conflates two different distinctions. The first distinction is between knowledge accumulation that involves

restructuring and knowledge accumulation that does not. Examples of the former include theory changes and novice-expert shifts. Examples of the latter might include learning the multiplication tables, the capitals of the 50 states and even much of the science that is learnt in school. The second distinction is between domain-general change and domain-specific change. Theory changes, and novice-expert shifts as well, involve restructuring but only of the concepts and explanatory principles of the domains of knowledge undergoing development. Such changes are domain specific. In contrast, the putative shift from preoperational thought to concrete operational thought are independent from particular content domains. (1985, pp.190)

Susan Carey has further pointed out that there exists an extensive literature that has shown examples of domain specific restructuring in the acquisition of knowledge and this is to be found in the field of History and Philosophy of Science. It can in fact be argued, that at the light of the analyses of the history of scientific enquiry conducted by the post positivistic philosophers of science, namely Kuhn, Lakatos, Feyerabend, it is currently accepted that the development of science proceeds in a discontinuous way. Within each domain of science, theory shifts can be seen as radical restructuring of knowledge. A fruitful analogy may therefore be drawn between the process of knowledge acquisition in the child and in the history of science. As we will see in the next sections Carey is not alone in attempting to draw such a parallel, and there are currently a number of researchers which have adopted the "child as a theory-builder" metaphor.

The last 25 years of Philosophy of Science have been characterised by an increasing interest in the diachronic or historic process of scientific investigation. According to this view scientific theories possess a developmental history which can be captured only by analysing theories in their dynamic process of transformation. The historical analysis of a particular body of scientific knowledge shows for Kuhn (1962), that explanations and theories do not proceed according to a linear increase in the accuracy of prediction and in the power of explanation. Theory change corresponds in fact to radical restructuring of theories which as Kuhn put it are analogous to "revolutions". To the cumulative view of progress in science, Kuhn substitutes the idea of revolutions i.e. the radical replacement of a scientific paradigm with a new one, as a consequence of the increasing number of anomalies which the old paradigm cannot account for. The history of science is characterised by a discontinuous process where a succession of paradigms substitute one another. As Laudan claims:

Theory transitions are generally non-cumulative, i.e. neither the logical nor empirical content (not even the confirmed consequences) of earlier theories is wholly preserved when those theories are supplanted by newer ones. (Laudan 1981 pp 141)

In the classical positivistic approaches scientific activity was viewed as consisting of three essential steps: the collection of observable data, the formation of scientific regularities or laws which are inductive generalisations from the observations, and finally the elaboration of theories as collections of these laws. Theories should hence consist of axioms, definitions, inference rules, and correspondence rules. Theories are tested once more by new observations which have the role of confirming or infirming hypotheses and predictions. Data is therefore seen as the secure basis for generation of laws and theories; within this perspective observations are in fact considered as basically objective and independent from theoretical biases. The inductive method was seen as the only real scientific method of inquiry, and logical formalism the most appropriate way of defining a theory.

On the contrary, Kuhn (1962), Lakatos (1978), Laudan (1977), Feyerabend (1978) among others, claimed that neutral data do not exist and that theories make sense out of observations and determine the meaning of the facts observed. Theories precede data collection and give perspective on what is observed. Science is seen as operating not within an inductive method based upon theory-free collection of data but in a theory-laden process that determines everything from the methodology to the actual observations which are possible. A theory makes sense out of the observations, it determines the meaning of what is observed and literally structures the possible observations. For Kuhn data is therefore fallible and each paradigm shift brings about a new observational realm.

There are obvious parallels that can be drawn between these assertions and Piaget's notion of the child's assimilatory schemas. For Piaget, an object is known only when assimilated to prior cognitive structures. Assimilation is an active process on the part of the subject and constitutes an "act of judgment" (Piaget 1950 p.68) Or in other words, assimilation is an act of inference or interpretation in which a datum is inserted in a set of concepts and gets its meaning from it. The nature of the structures the individual holds, determines somehow the aspect of the datum the individual will attend to. Within the two perspectives, genetic psychology and post-positivistic philosophy of science, observations of the world are always interpreted within a structuring system which gives them meaning. These systems can be cognitive structures, scientific paradigms or scientific theories. The essential point common to all is the existence of some form of structuring entity predetermining and pre-existing the observation.

Scientific theories are therefore organised systems of explanations which determine the nature of the observations and the methodology. The process of scientific inquiry, therefore, is one of constructing theories which embed methodologies, observational

techniques, deductive rules and rationales, which are therefore not independent from the theory nor applicable to all scientific disciplines. From this perspective, theories are not simple atomistic conjunctions of empirical laws but a structured whole in which there are terms and relations which are posited as hypothetical constructs and theoretical entities.

The proposal to consider that children construct "theories" of the world in a similar way to scientists can be seen as a way of conceiving of local structures which are liable of undergoing radical restructuring. In fact from the viewpoint of theory-formation the developmental process of "acquiring new knowledge" corresponds to a restructuring of knowledge as well as accumulation of new facts. The notion of theory reflects the idea of an organised, coherent and consistent system of explanation, which can underlie a number of strategies, explanations and predictions that the child will exhibit in his behaviour. It also provides an explanatory unit for cognition embedding some structural characteristics that distinguish it from other types of knowledge.

The first property of theories as explanatory units is that of describing local organisations of knowledge which constitute systematic unifications of different contents. Theories are the synthesis of a knowledge content and a logic which allow for the prediction of new states of affairs.

The second property of theories is their structural, holistic nature. They represent therefore a system of entities and relations that have an internal organisation specific to a level of development. Like in Piaget's structures, the addition of new elements to the theory creates a new set of relationships and thus new meanings.

From the holistic property derives the third property of theories, global restructuring from a theory to the next. The transition to a theory of a higher stage requires a radical transformation of the previous theory, therefore development is seen as a discontinuous process.

This process of restructuring is local and does not involve the totality of the cognitive system. Therefore subjects can be restructuring one domain of knowledge while remaining constant in other domains. The process of constructing new theories from old can thus be regarded as the creation of new abstract concepts and schemas that were not represented in the previous theory or were represented within a different system of relations.

2.3.3 The child as a theoretician: two proposals

The notion that children organise their knowledge contents in "theories" has appeared in developmental literature since the early seventies (Karmiloff-Smith & Inhelder 1975, DiSessa 1983, D.Kuhn 1989) and has since spread to related fields as educational psychology, learning and instruction (Driver 1983, Driver & Erickson 1983, Gilbert, Osborne and Fensham 1982).

By attributing to children the capacity of theory building clearly one is making a number of assumptions on their reasoning capacity: that children can entertain theories from early in development, that they can test hypotheses and draw inferences from them. A claim, that as we have seen, in post-Piagetian approaches is being currently made and empirically explored. The burden of the developmental explanation is thus put on the actual content of children's different conceptions. From this perspective, children of different ages do not differ in the kinds of concepts they can entertain but essentially in the explanatory, predictive power of their theories.

We will give two examples of approaches that attempt to explain conceptual development in terms of theory construction. Both Susan Carey and Karmiloff-Smith have given a central role to children's spontaneous theory building, although they differ in the scope they attribute to such theories and the emphasis they give on the actual process of theory elaboration.

2.3.3.1 Karmiloff-Smith

In the 1975 paper "If you want to get ahead get a theory.." Karmiloff-Smith & Inhelder analyse children's understanding of phenomena of balancing and equilibrium. They suggest that children's reading of the observations of physical phenomena depend on their implicit theories. Children are seen as constructing theories (in the specific task, theories of the balancing of objects); these are theories in action, i.e. they are not conceptualised. These implicit theories guide the child's actions and his evaluation of counterexamples, which in many cases are simply ignored. Children in fact, hold on to their theories as long as possible and when they first start taking notice of counterexamples they construct an independent theory before finally unifying all the events in a more general theory. Although the authors highlight the process of overgeneralisation that leads children to ignore counterfactual evidence this process is viewed as basically functional to the understanding of events.

It is interesting to note that this early statement of the theory-laden nature of the children's observations was produced in Geneva. This strongly supports the view that there is a continuity between the Piagetian idea of assimilatory schemas and the theory driven process of scientific discovery, we previously discussed. However an element of dissonance with Piaget's theory already appears in the article although very briefly and "en passant". This regards the capacity of concrete operational children to draw hypotheses.

The earlier claim (Inhelder and Piaget 1958) that "at the concrete level the child does not formulate hypotheses " must be reconsidered in the light of our children's tendency to act under the guidance of a powerful theory in action which involves more than observation of empirical reality. (Karmiloff-Smith & Inhelder 1975, p.209)

The authors suggest that in the simpler situations, in which subjects can devise strategies of resolutions, some form of hypothesis projection and testing is seen. For instance, young children predict that an object will always balance in its middle, they hence test their prediction by putting all the objects on the pivot in exactly the middle. The authors hasten to add however, that this is not to be taken as a capacity to conceptualise explicitly on what is being done.

Although Karmiloff-Smith has even in recent years given an important role to theory construction, this process has been inserted in a more vast framework of children's cognitive development, framework which appears to embed her original preoccupation with preoperational children's competence in hypothetical reasoning. Karmiloff-Smith proposes that many aspects of human knowledge are innately specified and initially organized in separate modules. However knowledge is initially procedurally encoded, fired by external stimuli and not available as data structures to other parts of the system. In subsequent development innate knowledge already stored is made available via a process of representational redescription. The process of redescription is an endogenous process whereby the mind exploits the knowledge it has already stored by re-representing recursively its own internal representations.

Subsequently in normal development, children ignore the external stimuli and focus on their internal representations. In other words, the normally developing child goes beyond successful output, to exploit the knowledge that he has already stored. (Karmiloff-Smith 1988b, p.30)

Once redescribed, knowledge then becomes explicitly represented and is thus available as data structures to other parts of the system and can be organised in theories. The particular nature of theories is that they are formed on the basis of

internal representations, while representations themselves are formed on the basis of innate specifications and from external input.

Thus the process of redescription occurs according to Karmiloff-Smith in three phases:

- phase 1, which is essentially a procedural phase, is characterised by the fact that children's behaviour is data driven. Children's procedures are success oriented and the processes they use are to attempt to match their present state with the goal-state. Much attention is paid to environmental stimuli, and there is no attempt to organise behavioural units into a consistent whole.
- phase 2, implicit knowledge is redescribed in internal representations. External stimuli are secondary, children go beyond the goal of success attainment and generate organisation-oriented procedures. The child now treats the previous isolated problems as one. In order to unify the preexisting elements of knowledge in a single representational framework he simplifies the procedures employed thus often incurring in error.
- phase 3, the conceptual phase is governed by a control mechanism which modulates the interaction between data driven and top-down processes. The child is in control of both his internal representations and the external stimuli. In contrast to phase 2 he can now take into account environmental feedback.

Taking the option of explaining development as the construction of specific domains of knowledge does not imply that there is no unitary mechanism of acquisition which recurs across all structures. A subject can be at different levels of organisation in two domains but within all domains the process of construction or evolution is the same. Karmiloff-Smith defends the view that different rates of growth can be compatible with a unitary process. She proposes a 3 phase model where the similarities across domains are not the consequence of "stage-like structures, tied to sequences of age ranges" but reflect the recurrent changes.

For this kind of change... I will use the term phase, with the hypothesis that children attack new problems by going through the same three phases, both within the various parts of particular domains and across different domains..... the recurrent phase concept argues for equivalence of each particular phase across domains. (Karmiloff-Smith 1986 pp.98)

From this sketchy outline of Karmiloff-Smith's approach, which does not do entire justice to the complexity of her framework, we can notwithstanding draw a number of considerations on the status she gives to theories in the child's cognitive architecture.

The theories which govern children's responses, starting from phase 2, are internal systems of knowledge unification and organisation. The process of representational redescription allows the child to go beyond successful action procedures, which are data driven, to internally reorganise and unify independent procedures. According to Karmiloff-Smith, the unifying role of phase 2 is not accompanied by test/falsify/modify procedures. Only at phase 3, are the theories flexible and children are capable of taking into account feed-back and modify their top-down procedures. At stage 2, the successful or unsuccessful responses are mediated by a theory which organises by simplifying the domain, and "carve and recarve" nature to satisfy children's predictions. Children in this phase, are far from the model of the inductivist scientist. Even later when they will be capable of modifying their theories to take into consideration some of their anomalies and the counterexamples encountered, it will be again a new approximation.

Karmiloff-Smith's contribution is also important on the empirical side, as she brings significant evidence on the process of spontaneous theory elaboration and the interrelation between theories and observations in the child (1984). Not only does she support the theory-laden hypothesis with a variety of data from children's problem solving activity but goes to show how at different moments in the construction of his theories the child interacts in a particular way with the environment.

2.3.3.2 Susan Carey

Susan Carey starts from the assumption that young children come equipped with the necessary competence to entertain and modify "theories of the world" Carey claims in fact, that human beings are theory-builders.

From the beginning we construct explanatory structures that help us find the deeper reality underlying surface chaos. (Carey 1985, p.198)

Children are seen as representing a few theory-like cognitive structures, which correspond to domains of knowledge such as, biology, mechanics, psychology or economics. A theory is characterised by the phenomena in its domain, its explanatory mechanisms, and "the concepts that articulate the laws and the representations of the phenomena". (Carey 1985, p.201) What characterises theories according to Carey is their explanatory power, property that relatively few conceptual structures embed. Thus in development children start with very few explanatory notions and build up more complex and articulated theories. For Carey the issue in cognitive

developmental psychology is to characterise the transformation of "core concepts" in conceptual change. Development consists in the restructuring of explanatory frameworks which are not domain general.

Cognitive development consists in the emergence of new theories out of the old ones, with the concomitant restructuring of the ontologically important concepts and emergence of new explanatory notions."(Carey 1985)

The studies Carey carries out show the emergence of new theoretical domains from old ones as, for instance, biology from psychology. The processes of restructuring that occurs in development are of two kinds, weak and strong: a weak restructuring involves new relations between the same set of concepts and of a strong restructuring involves change in the actual concepts held by the child. Theory changes involve this form of strong restructuring. One example of the strong restructuring is that of the differentiation of two concepts from an initially undifferentiated one. Wiser & Carey (1981) studied the undifferentiated concept of heat/temperature as it is found in the history of science before the time of Joseph Black and remarked that it has components that became specific to each of its descendants. However although some elements of the parent concept are to be found in the descendants, the parent concept occurs within a different theoretical structure than the following differentiated concepts and possesses a radically different structure. The same process of differentiation can characterise the transformation of some of children's concepts.

The interest of Carey's approach lies, in my opinion, in the status she gives to all of children's concepts. She claims for instance, that the concepts of the young child although undifferentiated, are true concepts as will be the later differentiated ones. These undifferentiated concepts are not to be seen as "confused" concepts, but as concepts that function within a theory as do all scientific concepts. By assuming that children as young as age 3, can represent true concepts and that there is no difference in the *kind* of concepts young children entertain with respect to older children or adults, her inquiry into the difference in knowledge between young and older subjects leads her to give a description of the theories that children of different ages entertain. Carey captures the difference in the content of knowledge with a description of the system of thought or theories that characterise and differentiate young from older children. Therefore, the analysis of the relation between concepts belonging to subsequent theories is carried out as a historical process. The task Carey sets herself is to identify those concepts that are coalesced, or undifferentiated or subspecified and trace their transformation in differentiated or specified descendants from the perspective of the theories they are embedded in.

From the two frameworks we have briefly introduced, it becomes clear that the notion of theory in cognitive development, comes to play the role of a local system of organised knowledge. Both models do not presuppose a change in the processing or logical capacity of the developing child but both propose a restructuring of the knowledge content in children of different ages. In my opinion, Carey (1984) makes a strong case for the view that restructuring can and does occur at the level of the explanatory principles underlying domains of knowledge, and that the sort of developmental change emerging in the differentiation or coalescence of concepts is evidence for such type of restructuring. She argues convincingly that theory restructuring is a deep restructuring that involves a number of concepts and the relations amongst them and thus it is a process that takes time (as is reflected in the discontinuity of cognitive development). This sort of restructuring does not necessarily have to involve a change in reasoning or representational power. Within a different perspective Karmiloff-Smith also provides convincing arguments supporting the view that the coherence of children's problem solving strategies, correct or incorrect, can be seen as underlying an organised explanatory system or a theory. The modification and progress of the child's understanding is characterised in terms of the restructuring of these coherent systems on new representational planes.

In conclusion, both Susan Carey and Karmiloff-Smith are willing to attribute the young child a certain amount of innately specified knowledge about properties of objects, about spatial relations, about language and number etc. They also agree on the assumption that young children can represent theory-like concepts from which they can derive predictions and new facts. However, although they differ in the description of the stages and the mechanisms responsible for their change, they both suggest that there is a radical structural difference in children's concepts at different moments in their development. Within any knowledge domain, from the initial conceptual structure held by the young child, to the more general and extended one of the adult, radical transformations take place that could only be characterised as the outcome of restructuring.

2.4 Conclusions

In this chapter I have presented some of the theoretical and empirical problems in cognitive development that constitute the background of my research. It is from these issues that emerges the theoretical framework I propose to account for the development

of some spatial concepts. I discussed an important intuition in Piaget's theory, namely that development does not proceed through linear accumulation of knowledge but through discontinuous stages which correspond to progressive restructuring of underlying knowledge structures. Abandoning Piaget's theory of general logical stages, I argued, does not imply going back to a cumulative model of knowledge acquisition. In fact, throughout the chapter it has been shown that a domain specific perspective, is compatible with a structural model of conceptual development, albeit the structures are defined at a local level. This entails a number of specific developmental predictions: alongside the independence in the rate of acquisition of different domains of knowledge, this approach suggests that in the development of a domain, the child moves through a fixed sequence of stages (without regressions) of greater internal coherence and problem solving efficacy. Furthermore, the process of cognitive development, of moving from a state of lesser knowledge to more knowledge does not appeal to a "second level" explanation of transformation in the representational format of the child, but solely by changes in the knowledge structure of the domain.

From this perspective the difference between a four year old and an adult is not in the logical format of the concepts they can represent mentally, but in the content of the explanatory conceptions they entertain. Furthermore as specialised structures are attributed to the child from the beginning, the question of the emergence of new concepts is shifted to that of the progressive transformation of available concepts. The locus of development is to be identified in the accretion of domain specific knowledge. The dimensions along which knowledge evolves are of the same nature as those of the development of science: greater explanatory power, predictive power and discrimination. The differences observed in children's responses to what appear to be similar tasks probing the same notion (as are the modified versions of Piagetian tasks) become indexes of the scope of children's theories.

Overall the chapter presented a discussion on general issues of cognitive development (the status of stages, the locus of developmental change etc.), and the advantages of a domain specific approach were outlined with respect to global empirical phenomena as heterogeneity across knowledge domains and early acquisitions. However a domain specific approach and in particular, a theory formation account of conceptual development, offer a specific framework to approach the development of particular concepts. In fact, in further chapters I will argue that Piaget's account of the development of the horizontal/vertical spatial coordinates suffers of some of the typical problems of his explanation of the development of concrete operational notions, namely the difficulty of integrating the behaviour of subjects which succeed at modified versions of the classical water-line task, nor the partial success in the task

itself. The reasons for Piaget's failure in characterising the development of this concept (as for many concepts of the preoperational period) are to be found in his theory of general logical operations. The domain specific perspective allows us on the contrary to integrate the replication data subsequently collected and provide a unified account of all the evidence available on the development of spatial coordinates. It does so by focusing on the change in content of children's concepts rather than on the representational prerequisites taken to be necessary to entertain a certain notion.

Chapter 3: A research strategy

3.1 Introduction

The research strategy I introduce in this chapter is organised around the simple idea that in order to investigate the transformation children's concepts go through in development, batteries of tasks probing the same concept should be employed. The general theoretical perspective that has been outlined in the previous chapter considers that the process of cognitive development consist in the gradual extension and generalisation of conceptual contents. The focus of this approach is on the gradual transformation of concepts rather than on their emergence at certain moments of development. The important question seems that of capturing how during development the same concept changes. This is radically different from attempting to identify the moment in which a certain concept appears and is made available to the child (a preoccupation compatible with the view that the child has to have attained a certain level of representational power to be able of entertaining a certain notion).

The distinction between charting the developmental transformation of a concept and identifying the conditions for its emergence, determines a parallel distinction between single critical tasks and batteries of tasks. While critical tasks are constructed to discriminate between subjects that have acquired a concept and subjects who have not, a series of tasks all probing the same concept can allow us to identify the conditions in which a subject can deploy his understanding of the notion under scrutiny. The new empirical question is that of analysing experimentally the different competence levels underlying the steps of elaboration of the concept. Therefore the objective of this type of strategy is to identify and explain orders and sequences in the solution of related tasks; sequences that mirror the progressive extension of the knowledge domain investigated.

There has been plenty of evidence from replications of classical Piagetian tasks showing how different versions of a task could elicit anticipated success. The modifications introduced by these studies were generally analysed in isolation by comparing the results at Piaget's task to results at a simplified version which was seen as allowing to emerge the true competence of the child. What is envisaged here is on the contrary, a *systematic* comparison of multiple variations of a task or in other words a systematic decomposition of the levels of children's conceptual elaboration in a specific domain.

The integration of a number of tasks is desirable to provide new elements for the interpretation of the theories underlying the performance at different stages. The more related tasks are analysed the more information on the type of problems a subject can solve at any one moment is collected, the more ground we have to make hypotheses as to the nature of the conceptions underlying the performance. Gopnick's (1988) article is a case in point. The theory formation account she proposes attempts to give a unified explanation of the infant's behaviour in a number of different versions of the original task of object permanence. Gopnick integrates a series of diverse experiments on the development of the object concept and produces some hypotheses as to the possible coherent theories that could underlie the different behaviours observed in infants of different ages. Her interpretation not only relies heavily on the coordination of multiple tasks, but does not elect any specific task as the critical task the solution of which discriminates subjects having acquired the "real" object concept. Each level is characterised as a particular theory of the object, at each level the infant has an object concept and there is no sudden emergence of the notion but simply a progressive refinement.

The chapter is organised in three sections. In the first I will discuss the idea of battery of tasks passed to the same group of subjects in opposition to critical tasks to assess competence in a specific domain of knowledge. I will then suggest that a within group analysis of the results at the battery of tasks as opposed to a between groups analysis is more informative with regards to the type of concept held at each stage of the development of a domain. Finally I will present a statistical method for within group analysis that is particularly suited to capture discontinuities in development.

3.2 Critical tasks versus batteries of tasks

The intuition behind the type of analysis I will be proposing, is that there are no single critical tasks to assess children's competence in a domain of knowledge. Rather than trying to establish when a certain concept emerges in development and therefore construct a task which "divides the sheep from the goats" I wish to investigate the transformation of the concept by observing which tasks a subject can solve and which he cannot. At each stage the child is considered having constructed a concept in its own right. Rather than seeing a concept attained only at the last stage and previous substages as mere steps towards the full blown concept, each successfully solved task is taken as evidence for the presence of a certain concept (although more limited than at the following stages).

The main consequence of the approach whereby a critical task discriminates between subjects who have acquired a concept and those who have not, is that no interpretation can be given to what type of conceptions are held by subjects who fail the task, nor to the type of conceptions leading to the "full blown" concept of subjects who demonstrate to possess it. The only index that is taken into account to establish what precedes the acquisition of the concept, according to Piaget's method, is the type of errors performed by incorrect subjects, e.g. by *via negativa*. If on the contrary one can find situations in which a child succeeds in applying the notion while still failing in other related tasks a much clearer picture can be drawn of his conceptions in the stages preceding the solution to the "classical" task.

If one is to push this line of thought even further, no task becomes the "classical" or "top level" task and the assessment of the progression in the child's construction of a concept, takes a fully diachronic perspective. A battery of tasks, probing the same notion, as for instance the series of object permanence tasks, chart the transformation of the child's understanding and usage of a certain property of objects. The problem is shifted from identifying the moment of emergence of a certain competence to characterising the conditions in which such competence is available at different moments of development.

If we consider Piaget's approach to the development of concepts at the preoperational and operational stages, there are tasks which can discriminate between subjects that have acquired the notion of cardinality for instance (conservation of number) and subjects who have not. A task such as conservation, or class inclusion, or even the horizontality task, all embed characteristics that according to Piaget require a certain level of logical coordination.

A typical example is the task of number conservation in which children must conserve the numerical equivalence of two rows of 7 objects after one of them has been changed of shape. The conservation task, is the task that discriminates for Piaget, between children who have an operational concept of number (e.g. they can represent the number of a collection and reason with it) and children who have a pre-operational concept of number (e.g. they have a pre-concept, or an intuition of number based on the spatial dimension and the configuration of a collection). The conservation task is a critical test of number concept in that to maintain the equinumerosity between two equinumerous collections, after a spatial transformation is performed on one of them, the child has to go beyond the difference in spatial size between the collections and consider the spatial transformation as independent from number. Complex inferences are involved in deriving the invariance principle. Inferences that imply reversibility of

the transformation, composition of the dimensions of the arrays, identity of the array before and after the transformation.

None of the other numerical tasks presented in the book "The child's conception of number" (Piaget & Szeminska 1941) according to Piaget, embed the operational complexity of the conservation task and are therefore solved at an earlier stage. With this I do not want to suggest that Piaget considered his tasks the *only* tasks to assess a certain competence, what I discuss is the idea that in order to attribute a subject with the competence or knowledge in a domain it is required that he express his conceptual ability within the constraints of that specific task. In most cases Piaget was ready to attribute a certain knowledge and understanding to the child only when it manifested itself in the most abstract form¹.

In an interesting discussion on early acquisitions or the effect of simplifying Piagetian tasks, Chapman (1988) argues that modified versions of tasks as conservation or transitivity as Bryant and Trabasso (1971) have transformed the classical situation to such an extent that the two versions are not comparable². Chapman claims that in many cases replications have transformed the task requirements in such a way that children can reach the solution through a form of reasoning that is not that required by Piaget's task. While Piaget investigated the operative reasoning that allowed subjects to seriate, for instance, the modified transitivity task could be solved by reasoning functionally or through correspondences. He concludes that the modified versions do not assess the same competence.

The problem I see in this argument regards the nature of the phenomenon one wishes to study, whether it is the acquisition of a certain type of reasoning strategy or the acquisition of a specific knowledge content. If the objective is to investigate the growth of a specific domain of knowledge, the question of whether a child is capable of using a certain notion (be it numeric, spatial or other), whatever the means to do so,

¹ I do not wish to enter in the discussion of false negative errors (Flavell 1977) and I consider that Piaget's tests do put an upper boundary to the evaluation of children's competence which is essential to take into account, however the strategy I will propose tends to spread out the assessment on a number of related tasks.

² Bryant and Trabasso (1971) studied the role of memory in transitive inference. Children were trained to remember the relative lengths of the adjacent members in a series of cylinders of increasing length. Transitivity questions had to do with the relative length of non-adjacent cylinders: is A smaller than C? etc. According to the authors this type of training allowed subjects to solve transitivity questions before they could solve the classical Piagetian task. Chapman argues that children may have solved the modified task not because the training on the premise relationships helped them remember the premises in making transitive inferences, but because the procedure used did not require memory in the first place but only required the encoding of some spatial clues.

is more crucial than the assessment of the type of reasoning employed. However Chapman does conclude with a remark in this direction:

..the weakness of Piaget's theory consists not in being contradicted by the facts but in lacking a means of specifying the task conditions that allow structures belonging to different levels (e.g. preoperational versus operational) to be used successfully. (Chapman 1988 p. 356)

In conclusion the method I wish to propose in alternative to the "single critical task" employed by Piaget and equally by most replication studies (that attempted to identify early acquisitions of operational notions), relies on batteries of tasks which all probe the same concept and that all share the same structure of the problem. Examples of such batteries can be the different versions of the object permanence, different versions of conservation tasks or in my specific case different versions of the horizontality task. By sharing the same structure of the problem I mean that all the tasks must for instance require a numerical evaluation after some type of transformation, or that all the tasks require recovery of an object after some sort of occlusion.

It is too difficult to discuss abstractly without concrete examples the criteria by which tasks should be chosen. The variations introduced are in fact relative to the concept investigated and depend on the preliminary hypotheses made with regards to the conceptions held by children at different stages. What makes a variation relevant and significant is determined by what types of theories the child is entertaining. For this reason it is essential to start with an evaluation of the existing replication data which can give a first insight on the type of transformation the concept of the child is sensitive to, and which therefore contribute to suggesting the preliminary hypotheses on children's theories at different stages.

In accordance with this perspective, I will design a *battery* of tasks where the same problem of representing spatial orientations is probed in a variety of contexts. The battery of tasks of the horizontality study described in following chapters, requires in every case to orient a surface of an object horizontally. In each task however, modifications are introduced which regard for instance the type of object or the spatial relation holding between the horizontal plane and the other parts of the display. Each version of the task is carefully chosen as to give some insight into the type of variables that affect the child's application of the notion.

3.3 What type of task?

The tasks that are envisaged in this method and those that will be employed in my experiments are all problem solving situations. I consider that problem solving tasks where the child is presented with a concrete material on which he has to perform some operation or reproduction, is one of the most informative methods to probe children's cognition. This, as opposed to strictly verbal tasks or yes/no judgements, or any other less interactive method. Problem solving situations as those used by Piaget for 40 years offer a number of advantages. The use of concrete material delimits the conceptual space of the problem, offers an interactional setting which allows a richness of observations that other type of tasks cannot provide.

Furthermore engaging the child in a situation in which he has to deploy his knowledge in order to obtain a concrete result, be it the reproduction of some aspect of the apparatus or the construction of numerically equivalent sets, allows us to examine the usage of a concept as it is applied to a specific context. In other words, this method complies very well with our objective of investigating both what a child is capable of doing with the knowledge he has at his disposal, and of defining the range of situations he can assimilate or cope with at different moments of development.

3.4 Establishing a hierarchy

Once we have designed a battery of tasks which all probe the same notion in a variety of contexts, (or recuperated a series of modified Piagetian tasks present in the replication literature)³ the first step consists in establishing the order in which these tasks are solved by a population of increasing age. Ideally all the subjects pass the totality of the battery Task 1 to Task n. Subjects' responses are classified as pass or fail. We will see that it is not always possible to pass a very extended battery to same sample, either because it is too repetitive for a subject to pass too many versions of the same type of problem, or because some tasks are not suited for the age range investigated (they may be too difficult or too trivial). A contingency table is then

³ The numerous replication studies of Piaget's tasks which by modifying some feature of the experimental situation elicited anticipated success, are a rich source of data on the type of variations that are worth exploring. It is in fact one of our objectives to reinterpret such data and give a unified account of classical and modified Piagetian tasks. In the experimental sections I will draw from some of the existing studies of the horizontality task which by modifying the apparatus have found earlier success than with the classical Piagetian water-level task.

drawn for each pair of tasks and frequencies are calculated of the number of subjects failing both tasks (f_{11}), passing both tasks (f_{22}), passing T1 and failing T2 (f_{12}), passing T2 and failing T1 (f_{21}).

		Task 1	
		Failure	Success
Task 2	Failure	f_{11}	f_{12}
	Success	f_{21}	f_{22}

Figure 3.1: Developmental contingency table

The type of scale I wish to draw from such a battery is independent of age and should define a dimension which may be called "extension of the concept". More important I wish to establish the relationship that holds between the different versions of the task, i.e. whether any two tasks are solved concurrently, or whether one task always precedes another. This is quite different from what Gopnick (1988) for instance does to relate the data on the object concept. Gopnick's unification of the different data relies in fact, on the age scale. Since different experiments were carried out on different samples, the concurrence of solution of two tasks is based on the fact that children of the same age solve the two tasks. I would like to introduce a more stringent criterion of concurrence and order by passing the two tasks to the same subjects thus establishing an order between the tasks based on the fact that one task always are concurrent with another (subjects either fail both or pass both) or always precedes another (no subjects fail T1 while succeeding T2). The latter case identifies what may be called collective *décalages* in the acquisition of related tasks, i.e. a delay in the solution of one task with respect to a second tasks, that has the same direction for all subjects (as opposed to individual *décalage*, where some subjects may acquire Task 1 before Task 2 and others Task 2 before Task1).

This more stringent criterion of order and sequential invariance allows us to establish a tighter relation between the tasks which can better sustain the interpretation of an underlying coherence in the children's responses. Coherence that is the characteristic of the theories organising their understanding.

The hierarchical analysis conducted on the battery of tasks should allow to draw the following type of diagram:

Stage 1 - T1 T2
Stage 2 - T1 T2 T3
Stage 3 - T1 T2 T3 Tn

Stages are seen as corresponding to the set of tasks solved concurrently. Transitions correspond to the periods in which tasks previously failed come to be successfully solved. The hierarchy is explained by the type of theory that can be attributed to the child at each stage.

Stage 1 - Th 1
Stage 2 - Th 2
Stage 3 - Th 3

For each stage, a characterisation of it will be provided in the terms of the underlying theory. This in turn will be tested by designing new tasks that under the characterisation proposed, should be compatible with the theory held by the child. For instance if I make the hypothesis that at stage 2 children can represent a horizontal plane only when this is perpendicular to its support, I can devise a problem that according to this definition should be solved correctly by all the children that also solve the other perpendicular situations of our battery. Should I find that this is not the case, I would have to revise my interpretation of what conceptions of horizontality is present at stage 2 when children can solve tasks 1 to task 3 for instance. Should the new task be found to precede or follow the stage 2 tasks, a new hierarchy will be drawn.

In conclusion two theoretical principles underlie this methodology:

1) the decomposition of stages in terms of the tasks solved. At every stage correspond a series of tasks that are solved correctly, every new stage represents an increase with regards to the previous one in the sense that new tasks are added as correct solutions. A hierarchy of tasks is established on a battery of tasks probing a notion.

2) the idea of a theory or a coherent conceptual organisation that unifies the child's solutions to different tasks. Underlying a set of concurrent successful solutions there is a theory which unifies the conceptual contents necessary to solve the different tasks. Each stage is characterised by a theory, therefore children's responses at a series of related tasks are assumed to be the product of a coherent system of beliefs. The question I will be asking when faced with the set of solutions of Stage 2 for instance, will be: what concept is held that could solve tasks T1 to T4 and yet fail T5.

Two empirical and methodological questions follow:

- how to establish an order between tasks and thus identify stages of development of a concept?
- how to test our hypotheses about the content of children's theories at different stages?

I will attempt to answer the two questions in order.

3.4.1 A statistical method for hierarchical analysis

Hildebrand, Laing & Rosenthal's (1978) Prediction analysis for cross-classifications proposes a statistical index (Del) to capture the discontinuity in developmental stage theories. It consists of a statistical method to quantify the extent to which a contingency table conforms to a triangular hypothesis of co-occurrence, or developmental order. The method gives a measure (Del) for each logically distinct prediction statement of the form: given that an observation has $X=x_1$, we predict $Y=y_1$. The Del is a proportionate reduction of error measure, which reflects the proportional improvement in the accuracy of the estimation based on the prediction over the expected frequency.

We present our data in a 2x2 contingency table, where rows correspond to success and failure in Task 1 and columns correspond to success and failure in Task 2. Each cell therefore corresponds to the combination of type of responses for the two tasks. The four cells are expressed as frequencies with respect to the total number N of responses (see table 3.1).

There are 4 patterns identifiable over the contingency table which indicate the order of acquisition of the two tasks T1 and T2:

- 1) concurrence: where all the entries are in the diagonal cells, T1 and T2 are both succeeded or both failed;
- 2) collective décalage: T1 precedes T2 - all entries are in the diagonal cells and below the diagonal i.e. either there is success in both tasks, failure in both tasks, or success in T1 and failure in T2. The cell corresponding to success in T2 and failure in T1 is empty.

3) collective décalage: T2 precedes T1 - all entries are in the diagonal cells or above the diagonal i.e. either both tasks are succeeded or they are both failed or T2 is succeeded while T1 is failed. The cell corresponding to success in T1 and failure in T2 is empty.

4) individual décalage - all four cells are similarly full. A comparable number of subjects succeed or fail in both tasks, succeed T1 while failing T2, or succeed T2 while failing T1.

The predictions are formulated as *triangular hypotheses*, for each of the models. The triangular hypothesis for concurrence where failure at T1 predicts failure at T2 and success at T1 predicts success at T2 and conversely, leads to expect frequencies approaching zero (within the limits of reliability of the response measure involved) in all non-diagonal cells. Prediction errors are those cases that falsify the hypothesis i.e. those that occur in the cells that are expected to be empty.

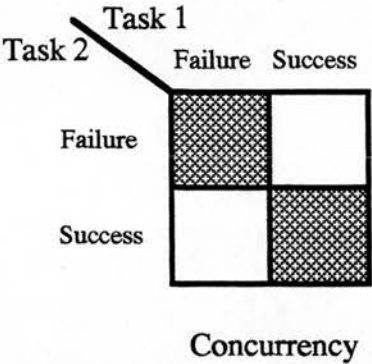


Figure 3.2 The response pattern of concurrence

The triangular hypothesis of collective décalage in favour of Task 1 where failure in T1 predicts failure in T2, while success in Task 1 predicts either success or failure in Task 2, expects frequencies approaching zero, in the bottom left cell corresponding to failure in T1 and success in T2. Prediction errors are those events that occur in the cell 2.1 expected empty, and falsify the hypothesis. On the contrary collective décalage in favour of Task 2 predicts that the top right cell will be empty.

		Task 1	
		Failure	Success
Task 2	Failure		
	Success		

Décalage $T2 > T1$

Figure 3.3 The response pattern of collective décalage in favour of Task 2

		Task 1	
		Failure	Success
Task 2	Failure		
	Success		

Décalage $T1 > T2$

Figure 3.4 The response pattern of collective décalage in favour of Task 1

When none of the cells are found empty and the frequencies are comparable in all cells a conclusion of individual décalage, is drawn. Children's responses distribute uniformly in the four cells, as some children solve both tasks, other children fail both, and for some children Task 1 is solved and Task 2 is failed, while for other children Task 1 is failed and Task2 is solved. This pattern is interpreted as indication of the fact that children can follow different paths when developing from one stage to the next.

		Task 1	
		Failure	Success
Task 2	Failure		
	Success		

Individual décalage

Table 3.5 The response pattern of individual décalage

The Del measure reflects the proportionate reduction of error against chance that is achieved by predicting certain cell frequencies of the table on the basis of the triangular hypothesis. The Del is computed as the ratio of the observed frequency of the error cell (s) and the expected frequencies for the same cell (s). If for instance, the triangular hypothesis predicts that cell 2.1 be empty, we compute the expected error by multiplying the unconditional frequencies of the cell (f2. and f.1) and dividing the product by the total number of observations. We then compute the Del by substituting observed error and expected error in the following equation:

$$\text{Del}_{2.1} = 1.0 - \frac{\text{obs.err. (f}_{21}\text{)}}{\text{exp.err. (f}_{2.}\text{f.1})}$$

This gives us a measure of the reduction of the error that one may expect under the unconditional probability. Del values range from zero to one. An index of zero suggests that the triangular hypothesis makes no improvement over chance. An index of 1 suggests that the triangular hypothesis provides considerable improvement over chance. Indices between zero and one, provide a measure of the proportionate extent to which the data support the hypothesis.

In order to determine whether an observed Del is significantly greater than a chance Del which is always zero, the standard error of delta is estimated. Once the Del index and its standard error are calculated, we can calculate a simple one-tailed normal curve test to determine the significance of the Del with respect to chance.⁵

A significant Del (at the level of $p < .05$) supports the prediction of the relative triangular hypothesis, that there are fewer subjects in the error cell than the marginal frequencies would predict. Similarly Del indexes associated with different triangular hypotheses on the same contingency table can be compared for significant differences.

Alongside the significance of a single Del against chance, we also rely on the difference between one Del and the indexes from the other triangular hypotheses.

⁵ The following equation defines the normal curve test statistic, z . For more details see Hoffman (1983 p.35). Estimation of variance is calculated by deviding the Del by the standard error:

$$z = \frac{\text{Del}}{S}$$

Significant differences between our hypothesis and the other possible triangular hypotheses, add support to the view that our initial hypothesis is the best predictor of order. Lautrey, Rieben & Ribaupierre (1985) argue that this second test of Del against the other two hypotheses relevant to order in a contingency table, introduces a second measure which neutralises the bias represented by using marginal frequencies. The second measure consists in balancing two Del indexes, one against the other, both of which were submitted to the same bias, while the first measure only tested the significance of a single Del against chance.

In summary, the procedure consists of several steps:

- we make an hypothesis as order is concerned, for instance we predict that T1 developmentally precedes T2 and therefore we predict that the error cell corresponding to success at T2 and failure at T1 will be empty;
- we compute the Del for this hypothesis of collective *décalage* in favour of T1;
- we estimate the significance level for this Del;
- if the Del is significant we determine if it is also significantly different from the Del indexes of two other possible hypothesis. The prediction is confirmed if the Del is both significant with regards to chance, and significantly greater than the other triangular hypotheses;
- if the Del is not significant, we calculate the other Dels for the alternative hypotheses and determine which prediction is significant, if any. The alternative models are therefore established post-hoc;
- if none of the predictions result significant, the hypotheses may be underdetermined by the data. Task1 or Task2 may be too easy for the age range and all the subjects succeed the tasks, as a consequence all frequencies are clustered in one cell and no order can be established. This situation identifies a problem in the sampling, the sample is too young or too old for the task. If on the contrary, the responses are evenly distributed in the four cells, we conclude by default that there is an individual *décalage* in the acquisition of the two tasks;
- if the Del corresponding to the prediction is significant, but not significantly different from the Dels of the alternative models (which themselves offer a significant improvement over chance), we conclude that *weak concurrence* underlies the acquisition of the two tasks. Since all three triangular hypotheses which expect the cells 21 and 12 to be empty are simultaneously verified, and no model is significantly better than any other, weak concurrence can be concluded. A special case of the previous situation is when the three Dels are equal to 1 (or close to). This indicates

that the three hypotheses fit perfectly the distribution. From this result, I conclude that a *strict concurrency* holds in the acquisition of the two tasks, as the corner cells 21 and 12 are empty (or close to be empty);

- if the Del is significant, but not different from one of the alternative models, both models are taken to represent the data. If one of the model is concurrency, then two models account for the order between the two tasks and coexist: *concurrency and collective décalage* in favour of one task. However, if the two models are of collective décalage, then the two tasks are acquired with *individual décalage*.

For this analysis I employed a statistical software that calculates the Dels, Z, and compares the Dels between triangular models, that was produced by Lautrey, Rieben & Ribaupierre at the University of Geneva. The program is compiled in Basic and runs on an IBM compatible PC. The program is not available on the market, but was kindly provided by the authors themselves.

3.5 Between subjects analysis

Although the accent of my analysis of children's performance will be on the results within subjects, a between subject analysis will be also carried out. Two related experimental designs are therefore combined in a within-subjects, with subjects nested in age-group, task condition, order of presentation, etc. design. The between-subjects component identifies changes in problem solving performance between age groups and discriminates between certain factors intervening in the experiment such as the order of presentation etc. The within-subjects component examines the patterns of acquisition orderings between task solutions. The latter design consists in testing models predicting performance in a task from performance in an other task for this the Hildebrand, Laing and Rosenthal prediction analysis will be employed. The former design results in the testing of hypotheses of statistical independence between groups.

The between subjects analysis is particularly relevant to test the difference between different age groups in the resolution of the tasks. The hierarchical method outlined above collapses all the age groups together. It is however important to examine differences in ages, both to consider the developmental progression and to allow us to target the tasks to the right age range. In fact if two tasks are presented to an age group for which they are too trivial or too complex, all the subjects will be concentrated in one cell, either all the subjects fail both tasks or they succeed in both. This result does

not allow us to apply the Hildebrand et al. test and no interesting order can be observed.

For the comparison of results between groups, when it will be possible (i.e. when the data is measured at the ordinal level) I will be relying on standard parametric statistical methods i.e. Analysis of Variance as available on BMDP statistical programs. On the contrary when the data will be measured at the nominal level, performance measured as correct and incorrect responses to the task, I will rely on standard non parametric statistical methods i.e. the χ^2 test and the McNemar test for significance of change in related groups.

3.6 Testing hypotheses about theories

The same method is used for two different phases of the research: establishing a hierarchy between existing tasks, and testing explanations provided for the hierarchy by making hypotheses of the order which relates new tasks to previous ones. It is in fact, this second phase that is crucial in the empirical procedure as it determines the validity of the explanations we provide for the order in which the tasks are solved in development. For instance, once we have shown the hierarchy that holds between modified replication studies, (which is done with no strong a priori hypotheses on the order relation that holds between the existing replications) and provided an account of the underlying competence levels and conceptions characterising the various stages identified, a new set of tasks is designed to test this account. If for example, children pass Replication 1, Replication 2 and Classical task, an order of solution of the following kind may appear:

Stage 1 no task solved

Stage 2 Repl.1 and Repl.2

Stage 3 Repl.1, Repl.2, Classical

From this picture, a number of explanations can be provided for the orders identified and hypotheses can be drawn as to the concept held at each level. On the basis of these hypotheses, new tasks can be designed probing the same notion, to test the interpretations. It can be assumed for example, that at Stage 2 a child's theory will allow him to solve a Task 3 which is regarded as requiring knowledge compatible with the theory he is currently holding. Therefore I will make the hypothesis that Task 3 will be solved concurrently with Replications 1 and 2; that the triangular model

between Task 3 and Repl.1 will be one of concurrence. In this case the Hildebrand et al. method will be used to test a precise order relation. If another type of relation is found, for instance Task3 preceding Repl.1, the original interpretation of Stage 2 must be reconsidered and an alternative explanation of the child's concept at stage 2, and for the reasons that make Repl.1 a more difficult task, will have to be produced.

The strategy relies on alternating between establishing a hierarchy, drawing hypotheses and designing new tasks, and testing the order relation that holds between the new task and the previous ones. Initially a re-examination of existing data will be carried out: a series of tasks from the literature will be presented to subjects as a battery. All the subjects will pass all the tasks and an order between the solution to the various versions will be established. A picture of the kind described above will be elaborated: stages will be determined according to tasks solved concurrently. Then an analysis of the tasks, of the modifications they introduce, of the variables they manipulate, will be conducted, with the objective of making a number of hypotheses as to the nature of the conceptual systems that at different stages can cope with the different problems.

Concretely the research strategy will be as follows:

- identifying and defining the notion which will be studied, e.g. horizontality
- analysis of Piaget's original study and data description
- review of the relevant literature to establish which variations were introduced by replication studies, and in particular what modifications elicited anticipated or retarded solutions,
- designing of a small battery of tasks to compare performance of a subject in a number of versions of the task, in an attempt of constructing a systematic hierarchy in the solution of the various tasks (modified and classical)
- hypotheses produced as to the type of theories held at each stage
- new tasks designed that probe hypotheses
- a new hierarchy produced with addition of new tasks

3.7 A historical precedent

It is interesting to remark that a hierarchical method was proposed in the sixties as a method of testing Piaget's hypothesis of the invariance in the order of acquisition of concepts of different operational level. Wohlwill writes in 1968:

There is a surprising paucity of empirical evidence bearing on this uniform-sequence question. If the developmental sequence can be formulated in terms of a set of hierarchically organised tasks, then an examination of the response patterns made by a group of subjects to these several tasks will yield information of direct relevance to the question of sequential uniformity. Reduced to its barest essentials, this situation might be represented by a two-by-two contingency table, indicating frequency of success and failures for any pair of tasks. For the postulate of sequential invariance to hold, it would be necessary for the frequency of one of the cells to approach zero, within the limits of reliability of the response measures involved. This would be the cell representing the combination of success on the supposedly higher level task and failure on the lower level task. (1963 p.256)

There were a few attempts (Keats 1955 for instance as quoted by Wohlwill) to probe the order of acquisition of Piagetian tasks with this method. In the same line, Wohlwill suggests that Guttman's scalogram analysis could be applied to a series of Piagetian tasks. Guttman's (1950) scalogram analysis simply portrays the number of responses in a sample that conform and fail to conform to a particular ordering, without yielding a value for the probability that the items form a scale in the population sampled. Guttman proposed that the degree to which a sample of responses approximates a unidimensional scale be assessed in terms of a coefficient of reproducibility which consists of the proportion of responses in the sample that can be reproduced from an overall ordering of subjects that most nearly approximates a uniform order. However Wohlwill argues that the typical Piagetian sequence of stages rarely comprises a sufficiently large number of definable steps to allow a precise sequence to emerge. Moreover Piagetian sequences typically involve a succession of mutually exclusive types of responses to one and the same task, rather than a set of cumulative responses to a series of tasks.

Flavell thus refers to the study by Kofsky (subsequently published in 1966) who designed a series of eleven tasks to probe the developmental sequence in the domain of classification. However Kofsky was able to use only partially the sequence described by Piaget. The problem Wohlwill identifies is that in fact of applying a hierarchical analysis directly to Piaget's stages.

Under certain circumstances it may be possible to construct a series of cumulative tasks so as to tap the various response types suggested by Piaget. More frequently, however the resulting sequence provides instead a measure of the generalisability of a given response along a graded series of problems. This, paradoxically is not a question which Piaget has handled very systematically. (Wohlwill 1963 pp. 257)

The problem Wohlwill highlights is that of the sequential dependence between stages and he concludes that to trace prerequisite relations between concepts would also need other techniques as longitudinal studies. The type of predictions that Piaget makes



with regards to the order invariance rely in fact on the logical relations between concepts of different order. Thus transitivity is a logical and psychological prerequisite for measure. In order to establish the functional necessity of a specific sequence therefore, probably does require additional validation

The conclusion Wohlwill reaches is encouraging with regards to the application of such a type of methodology. As we pointed out previously the hierarchical analysis is intended to highlight the order in which tasks of the same domain are solved with the underlying assumption of a progressive generalisation of the concept to different contexts. The hierarchical method address the issue of specifying the parameters and variables of specific contents which can affect the developmental timetable with respect to a particular concept. The prerequisite relations that I will try to establish will not relate concepts of different level (operational or other), but rather different contextual situations in which the concept comes to be applied. Therefore when Wohlwill claims that the sequence analysis is apt to capture the generalisability of a response to graded problems he is in fact describing precisely what I wish to obtain.

Chapter 4: The development of horizontality

4.1. Introduction

The theoretical and methodological system that has been outlined in the three previous chapters will now be applied to the study of a specific phenomenon of cognitive development: the development of the horizontal/vertical system of spatial coordinates.

The study of the development of spatial reference systems was first introduced by Piaget and Inhelder in their book "The representation of space in the child"(1947). As for many topics in child development, Piaget provided the background for most of the subsequent research in this field. Piaget designed a task to probe children's understanding of the horizontal which is undoubtedly very ingenious. He showed children a bottle half full of water, tilted it at various angles and asked subjects to draw the water level for each of the different orientations. Piaget chose this setting to examine if children were able to make use of an external reference system because there was no fixed relation between the water-level and the sides of the container. The free surface of a liquid is always horizontal regardless of the position of its container. The surface can be said to be independent from its container because of the dynamic properties of liquids. Thus, children could not simply conserve the spatial relations between the elements of the display across all modifications of tilt, but had to rediscover every time the relative orientations of each element. This could be done only by referring the position of each element to an external frame of reference. Piaget concluded therefore, that if subjects can correctly represent the orientation of the water for various tilts of the bottle, they are using a vertical/horizontal frame as a reference. In sum, the water-level task represented for Piaget, the critical task to assess the acquisition of a stable system of horizontal reference.

I have chosen to investigate the development of spatial referents, the horizontal-vertical system of coordinates starting from Piaget's task of the reproduction of the water-level, for three main reasons:

- 1) the incapacity of children until the late age of 10 or 11 to correctly represent the orientation of the water surface in a tilted container, is a solid empirical phenomenon often replicated, it has in fact become the conventional means to assess children's acquisition of horizontality;
- 2) Piaget's explanation of children's errors in the long period going from the first attempts to draw the water surface with a line to the final correct reproduction, remains incomplete and in my opinion insufficient;

3) the existence of such a long intermediate or preparatory phase, which has been left relatively unexplored and unexplained by subsequent researches in the field of spatial cognition, cries for a more detailed analysis and interpretation of children's understanding of spatial reference systems.

In other words the characterisation of the development of the coordinate system suffers some of the problems of Piaget's account of the pre-operational stage I have discussed in Chapter 1. The problems arising in the interpretation of the results at the water-level task are in fact, directly related to the problems of the general theory of operational thought. There is first of all, the difficulty of integrating the data from subsequent replication studies showing partial or context bound success to tasks sharing similar structural constraints to the original task. The issue of early acquisitions that was discussed in general in Chapter 1, finds here a specific exemplification. There is the problem of the abstract nature of the concept that is seen as the product of the developmental process, an issue that is raised by the data that has shown a considerable number of adults failing the water-line task. Finally there is a lack of detail in the analysis of the intermediate phases, which account for 5 to 6 years of development (from the age of 5 to the age of 11). What do children know about horizontality, how is their spatial representation organised in the long period before they succeed in the water-level task, remains unclear in Piaget's account. Here again we find a special case of the general problem of Piaget's limited account of children's knowledge during the pre-operational stage.

The theoretical framework that is outlined in the first three chapters, was conceived to address some of the problems emerging from Piaget's theory of general structures. The framework should thus help us to approach the problem of analysing and characterising the development of the horizontal and vertical coordinate system. The perspective of interpreting the change in children's conceptual development without appealing to domain general principles of cognitive organisation, but solely from within the specific domain (i.e. as a content specification), can provide the necessary background to explore the knowledge held by children in the long intermediate phase described by Piaget. I will in fact, attempt to provide a finer grain analysis of children's understanding of the notion taking into account all the data showing context bound or partial success in the children's capacity to represent the orientation of horizontal planes. By concentrating on the coherence underlying each stage of development of this notion (and in particular of the stages in which subjects fail the water-level task), a new interpretation of the developmental process can be proposed. The chapter is organised in three main sections:

- presentation of the task of the water-level as was first introduced by Piaget, report of Piaget's results and overview of Piaget's interpretation of the development of the notion of horizontality,
- a review of the replication studies,
- a general discussion of all the data reported on the water-level problem and analysis of Piaget's theory of the development of spatial coordinates at the light of the evidence from the replication studies.

4.2 Definition of horizontality

Before presenting in detail the experimental paradigm that Piaget introduced for the study of the development of the horizontal/vertical system of coordinates, it seems necessary to provide a definition of the concept under scrutiny. The notion I will be examining in the next few chapters is a spatial relation, the relation of horizontality. To say that a physical object is horizontal is to consider that it lies on a plane perpendicular to the direction of gravity and parallel to the line of the horizon. The horizontal plane can be tangent to the earth's surface in a point, or parallel to the tangent plane. Geometrically a horizontal plane is parallel to the horizontal axis of a system of orthogonal cartesian coordinates. A system of coordinates is a reference system in the sense that its elements are supposed unchangeable and immobile. The system of horizontal/vertical axes are the attribution in the physical world of the mathematical concepts of frame of reference or system of coordinates. Thus horizontal and vertical are both physical concepts expressing the direction of gravity and the perpendicular to it, and are mathematical abstraction of these physical notions. Strictly speaking on earth verticals are not parallel and horizontal planes are curved. Geometrically however the discovery of the vertical and horizontal leads to the construction of a system of perpendicular axis representing the physical state only as a theoretical approximation.

Conventionally planes such as table-tops, ceilings and floors are taken as references for horizontality in our daily experience. This is because the physical realisation of horizontality relies on the property of being perpendicular to the direction of gravity. Objects that are manufactured to be supports for other objects, i.e. keeping them in a state of equilibrium (thereby avoiding that the supported objects fall or roll off) are designed to counteract the pull of gravity with an equivalent opposing force which goes in the opposite direction of gravity. They therefore have to be perpendicular to

the direction of gravity in order to oppose a force which has the opposite direction of gravity and nullifies the pull along the surface.

In conclusion the horizontal is both a physical and a geometrical notion. Physical in the sense that planes perpendicular to the direction of gravity have been chosen for all the consequences that this property has on the statics and dynamics of objects. Spatial in the sense that it is an abstract system of reference that organises our cognitive space as an empty container within which positions and orientations of objects are related.

4.2.1 The physical properties of liquids

It was mentioned above, that in our daily experience the reference for horizontality is chosen from our direct environment, as for instance the floor or table tops. Other horizontal planes that are conventionally used as reference are liquid surfaces at rest. Regardless of the orientation of its container the free surface of a liquid remains always horizontal. This is so because of the physical-dynamic properties of fluids. As most of the research on children's representation of horizontality requires subjects to represent the orientation of a liquid surface, I will briefly remind what are the properties of liquids which determine this phenomenon.

A quantity of liquid has no shape of its own. We say that a portion of matter is a liquid when its external surface can be easily deformed by the action of small impulses. When a quantity of liquid is deformed it remains all in one piece and its volume stays constant. The cohesive forces that exist between molecules keep them in contact and prevent them from becoming dispersed. However the thermal motion of atoms in the liquid state is too great for the binding forces to maintain the molecules in a permanent and well ordered structure as they do below melting point. A close packed coherent assemblage is formed but the molecules are not prevented from "sliding" over each other.

The shape a liquid takes in a container when it is in equilibrium is determined by two factors: surface tension and gravity. Every molecule at the surface of a liquid is subject to the attraction of its neighbours but, unlike one in the body of the liquid, all the forces are directed toward one side only. There is thus a resultant directed towards the interior and this gives rise to a pressure round the exterior as if the liquid was surrounded by a stretched elastic membrane which naturally tends to reduce its area. On the other hand there is the effect of gravity. Liquids are weighty because as all matter they are subjected to earth's gravitational field. Just as a movable solid is in equilibrium when its centre of mass is as low as possible, so the equilibrium shape of a

quantity of liquid is that which has as low a centre of mass as possible, taking into account any constraints on it. For this reason, a liquid occupies the bottom of the vessel that contains it and is limited by an upper horizontal free surface. The shape of the liquid is determined by the combined effect of two forces, mass and surface tension. The modification by the feeble surface effects is in fact, very slight and makes the free surface of a liquid not strictly horizontal but bound at its edges by a meniscus which may be either convex or concave. In conclusion, the characteristic orientation of the surface of liquids is due to two factors: one arises from the structure of the liquid and its deformability; the other arises from external circumstances and is the existence of the gravitational field.

4.3 Piaget's study of horizontality

4.3.1 The water-line task

The experiment observes children's estimate of the horizontality of liquids from the age of 4 to 11. Subjects are presented with a bottle half filled of liquid and a picture representing the side outline of the bottle and are asked to draw the water. The bottle is tilted at various angles and every time a new drawing is requested. By modifying the orientation of the bottle, Piaget & Inhelder observed how the subjects discovered and represented the constant horizontal position of the water surface.

The equipment employed in the experiment is the following:

- a square bottle and a round flask filled to a third with coloured water and hermetically closed,
- 2 bags of cloth in which to hide the bottle leaving the cork visible
- a few sheets of paper on which are drawn 8 different positions of the bottle; the drawing consists of a side outline of the bottle sitting on a horizontal line suggesting the stable horizontal support,
- the angles at which the bottle is oriented in the drawings are: 0%, 45%, 90%, 135%, 180%, 225%, 270%, 315%.
- a stable horizontal support, coloured pencils.

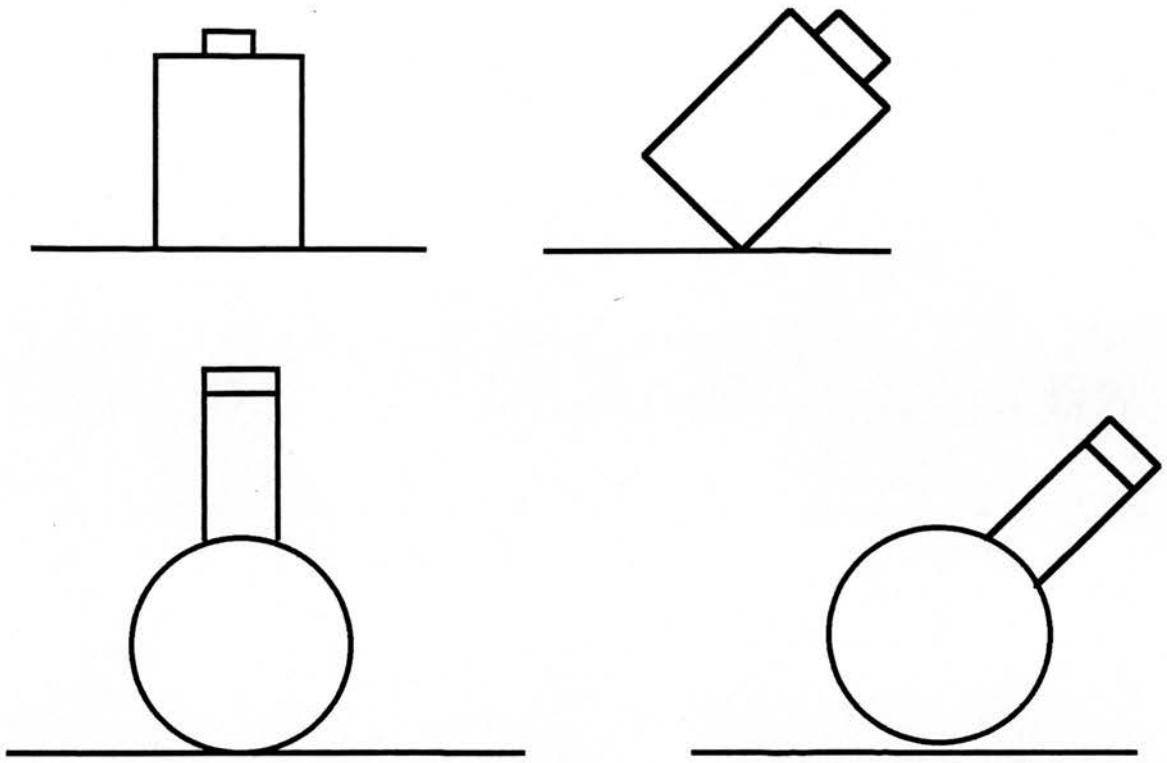


Figure 4.1 Side outlines of the square and round bottles in the upright and in the 45 degrees position.

The standard procedure is the following: the experimenter will show the child the bottles, tilting them at various angles. In the *anticipation* situation the bottle is put in the bag, leaving the cork visible and then is rotated to a certain angle. The child is given a drawing of the bottle tilted at the same angle and is asked to draw the water line when the bottle is in that position. This is repeated for all 8 positions. From time to time the experimenter will ask the child to explain why he has drawn the line in a certain way, or to describe verbally how he imagines the position of the water. The younger children (under 5 years of age) who cannot draw lines are asked to indicate the water line on the bottle with a gesture.

When the subject has finished drawing all the positions, the bottle is taken out of the bag and a new set of drawings are handed to the child. This time he is asked to draw the water line for the various inclinations of the bottle, with the bottle and the water directly visible in front of him. This is called the *copy situation*. The experimenter tells the child to draw the water as he sees it. Another set of drawings may be asked with the bottle back in the bag, to verify the effect of the copy phase on the child's anticipation of the level. Finally the child is shown all his drawings and is asked if

there are any corrections he would like to make. The task is then repeated for the round bottle.

Piaget & Inhelder occasionally complemented this method by others. For instance by giving the children prepared cards with the water line already drawn on them, from which children selected the correct one. Alternatively the authors used paper cut outs representing the bottle and the water, and asked the child to place them so as to reproduce the correct orientation. Of course variations in the verbal questions could also occur in the spirit of the clinical method. The method implies in fact, a great attention to the subject's direction of thought and requires the experimenter to be partially directed by the subject's own actions and reasoning.

4.3.2 The behavioural stages.

The situation outlined above gives rise, according to Piaget & Inhelder, to 3 stable behavioural patterns corresponding to 3 stages of development.

STAGE 1 - the child is unable to represent the water as a plane surface. He draws the water as a scribble, a little ball, or a dot. The only spatial relations that the child seems to be representing, are topological ones and in particular the relation of inclusion. The child represents the water as something inside the bottle, and not in terms of straight lines, planes or inclinations. There is a general disregard for the orientation of objects. This stage lasts until the age of 4 or 5.



Figure 4.2 Stage I errors in the reproduction of the water-level

STAGE 2 A - Children at this stage draw lines to represent the water surface. The striking observation is that the water line is drawn parallel to the base of the bottle for all orientations. This error is performed in the anticipation condition but also in the copy situation. In the latter case in particular, regardless of the evidence the child has in front of his eyes, he will draw the water parallel to the base. No amount of counter-suggestions, like asking the child to put his finger on the bottle at the level of the water

and then follow the rotation of the bottle, will enable the child at this stage to draw the line correctly. Occasionally the drawings seem to represent an expansion of the water toward the beaker, as if the volume had increased, in order to reach the beaker.

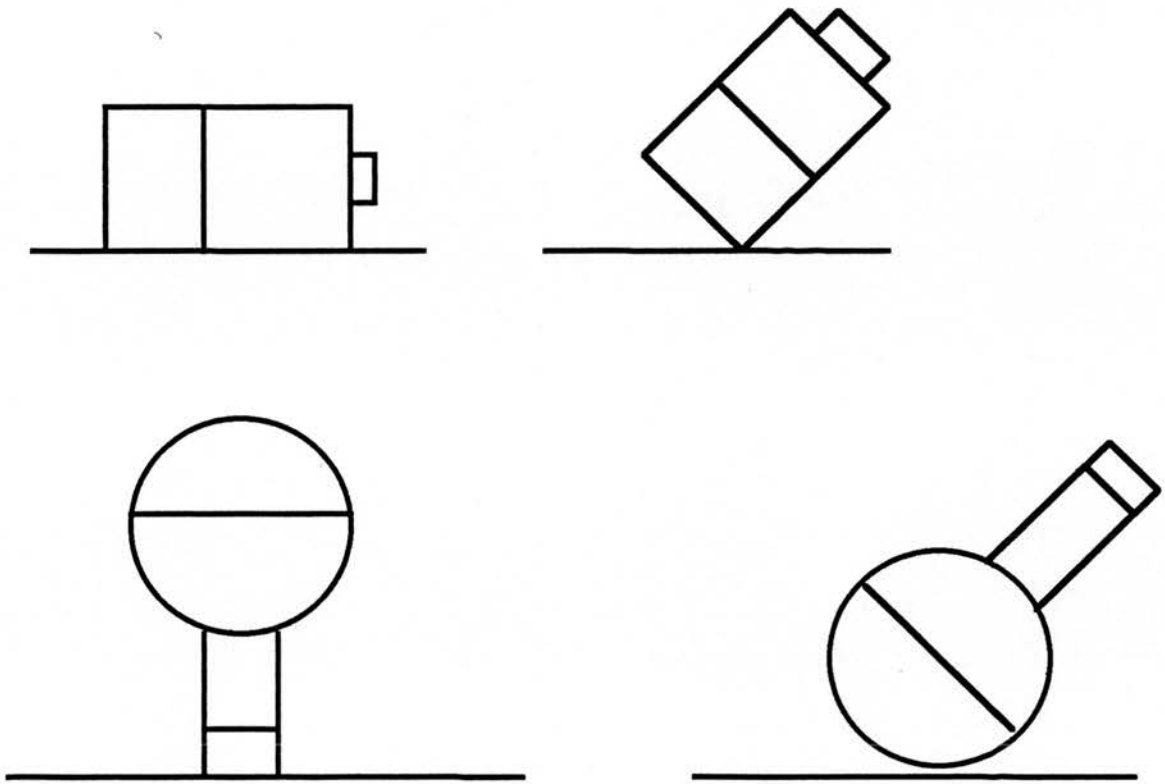


Figure 4.4 Stage IIa errors in the reproduction of the water-level.

They constantly assert that the water remains parallel to the base of the jar, and the reason they do so, is simply that they continually repeat what they observed with the bottle in its original position and fail to note or deduce correctly the subsequent positions. Piaget (1947 p.389, my translation)

Piaget argues that the improvement of these children with respect to Stage 1, not only consists in their novel ability to draw planes, but also in their capacity to refer the surface to another object, namely the base of the bottle. For Piaget this indicates the lack of an organised co-ordinate system; it shows however a beginning of a concept of parallelism even though based on the wrong reference axis.

STAGE 2A intermediate - This substage is characterised by responses which are intermediate between the gradual discovery of the horizontal and the mistakes of the

previous stage. The children here can anticipate that the water will move toward the beaker showing by a gesture or in verbal terms, that the water will be higher on one side than the other. Their pictures however will not reproduce their verbal descriptions and will be very similar to those of the preceding stage (parallel to the base, water hanging to the top when bottle is up side down etc.).

STAGE 2B - the subjects at this stage no longer draw the water parallel to the base, they regard it as moving relative to the jar. However their lines are not horizontal, but oblique, running from one corner of the bottle to the opposite. The water is seen as mobile but the children still fail, according to Piaget, to relate the orientation of the water to an external frame of reference. Children's comments are of the sort: "there will be more water on one side" "water goes towards where it is tilted". All the drawings described by Piaget show tilted oblique lines. The age of this substage covers the period from 5 to 7 years.

Fis (7.6) - is interesting for his reactions after the experiment. Before seeing the water in the tilted bottle he reacts as all the others: "The water will be tilted (draws water level with lines starting from the bottom corners of the bottle)". Of the two drawings presented to him, he choses the tilted one (the other one represented the water horizontally). "Now lets look if your drawings are correct. Check with this ruler on the bottle." - "It is wrong"- Why - "Like this it stays straight." "And if we tilt the bottle" - he tilts the ruler- "I think you should leave it straight" - "No it is impossible" - "Look" - "Ah yes"- "And if I tilt it more?- "It will be higher" he tilts the ruler. (1947 p.342, my translation)

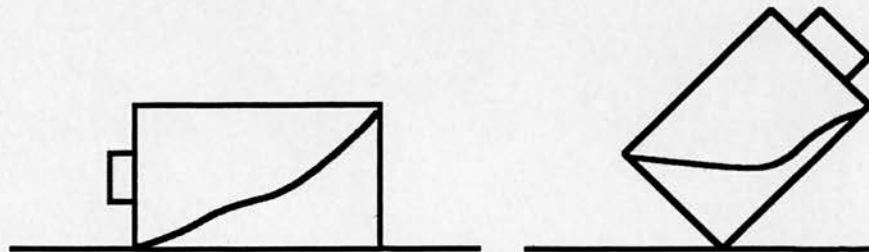


Figure 4.4 Stage IIB errors in the reproduction of the water-level.

STAGE 3- this stage marks the attainment and generalisation of the horizontal. It is a progressive discovery rather than a sudden complete acquisition. It covers an age range of 7 to 11 years. The behaviour classified by Piaget is the following: some children start by discovering the horizontal for all the orthogonal inclinations 90%, 180%, 270%. Only subsequently do they generalise to the oblique positions.

For others, Piaget calls them stage IIIA, there is a trial and error construction of the horizontal for all orientations; these children may begin by drawing the line tilted but after observation of the display and a few trials, the horizontal is recognized and applied to all the positions. The interplay between predictions and observations helps the subject to realise the invariance principle.

Fro.(9.6)- He starts by drawing all the lines oblique, then when looking at his lines tilted 20 or 30 degrees, he cries out: " No my drawing is too tilted, water can't be tilted. It is always straight, because water must be straight". (1947 p.344)

Finally at substage IIIB the horizontal is solidly generalised to all positions in anticipation as in copy. The children represent the horizontality of the water-level from the beginning and have no need to observe the display to anticipate the water's position. Their verbal comments seem to imply a good understanding of the behaviour of the water, dynamic as well as geometrical.

4.3.3 Piaget's explanation of the development of horizontality

For Piaget the discovery of the permanence of the orientation of the surface of liquids depends on children's ability to relate the position of the bottle, the position of the liquid and the environment. Hence, Piaget's explanation of children's behaviours at the water-level task relies on the more general theory of the role the vertical/horizontal coordinates play in spatial cognition.

The importance of the horizontal/vertical system of coordinates in the cognitive organisation of space, was stressed by Piaget in his studies on the development of spatial representation. A system of reference is essential to organise our spatial relations because it fixes a point from which all orientations can be referred to. How is it possible, in fact, to conceive of a direction or a position of an object without situating it in a spatial environment where specific points have been chosen as references? Piaget considers that the function of a system of reference is to structure the relations between objects but also to define space as a container which has identifiable points and measurable distances.

To borrow Piaget's terms, the coordinates of the euclidian space create a vast network extended to all the objects, consisting in order relationships applied to the three dimensions. Every object situated in this network is hence, coordinated to all others according to three types of relations: right/left, on/under, in front/behind.

However a system of coordinates, is not simply a network of order relations between objects; it applies to positions as much as to the objects in position...., it constitutes the euclidian space as a container independent from the mobile objects it contains. (1947, p.386)

In other words psychologically, the problem lies in the coordination of a system of relations including all objects such that any object can be related to another in terms of position and distance. The axes of the coordinate system are in fact privileged reference points from which all objects, positions and distances are determined. They themselves are relying on particular objects kept immobile by hypothesis which in the mathematical world is a point chosen as origin, and in the physical world are the horizontal ground or the vertical objects that are perpendicular to it.

In order to solve the task correctly, the subject must relate the position of the water to the position of the container which itself is related to the table or a wider environment. The coordination of these orientations are none other, claims Piaget, than the coordinations of relations between objects which is at the base of euclidian representation of space. The orientations of the various objects have to be related to some privileged point or axis, chosen as reference for all of them. This allows the subject to determine the position in space of the various objects and hence reproduce their relative positions on the smaller space of the page. The privileged axes are the horizontal and vertical coordinates given by the natural environment.

The behavioural stages identified, reflect the process of coordinating more and more elements, and positions, and relating them all to a fixed reference chosen for its independence from all displays:

- Initially the only spatial relation represented correctly is that of inclusion, the scribbles of the very young subjects are accurately centered within the boundaries of the outline of the container. It can be recalled that this is an improvement over yet younger children that scribble all over the page and are not constrained by the boundaries of the container.
- Children that are incapable of identifying the coordinate axes, refer the water-line to lines "internal" to the space of the container and draw the surface parallel to the base of the bottle. Piaget claims that these subjects are lacking the operational capacity necessary to relate all the elements of the display and can only center their attention on the proximal relations internal to the bottle. Piaget interprets the children's reliance on the proximal entities as the persistence of the topological spatial representations of previous stages, which only takes into account relations as proximity, inclusion, or closure and thereby only accounts for the direct relations between objects.

- Later, the child will know that water "goes towards the beaker" and will represent this with oblique lines, which indicate again the inability to choose the correct reference plane. The child, in fact, after having abandoned the idea that the water is always parallel to the base of the bottle still does not know what to refer the water line to; hence he relies on the opposite corners of the bottle and simply draws a line between them.
- Finally, the subject can determine the orientation of the water surface by relating it to a frame of reference external to the container. As the situation deals with physical objects in the world, the frame of reference is the natural system of horizontal and vertical coordinates with which we organise our natural spatial environment. All the elements of the display are considered and the spatial relations between each element are coordinated. The child can finally conceive of an empty space seen as a container in which all these objects and positions are situated.

In conclusion, for Piaget, only when the child has related the orientation of the water to the frame of reference, will he know that the water is horizontal and draw it parallel to the other horizontal elements of the picture. In other words he will have coordinated all the elements in the display within a three dimensional space organised as a grid of orthogonal relations.

4.3.4 The operational system underlying the horizontal.

According to Piaget, this account of the development of spatial coordinates was not only supported by the data collected from the water-level, but was also supported by a variety of other experiments in the spatial domain. These results taken conjointly pointed toward a gradual construction of the concepts that make up an euclidian theory of space. The euclidian relations which only appear at Stage 3, allow in fact, the solution of a whole new set of problems that were not treatable in the topological system of the previous stages. Without attempting to summarise the 30 or more experiments presented in the "The child's representation of space" I will briefly recall two studies on the vertical.

The better known task on representing vertical orientations, consists of anticipating and drawing, the position of a plumb line hung to a frame, when the frame is tilted in different directions. Piaget reports errors in this task which match closely the errors at the water-line task. Here as there, there is an initial attempt to keep a fix relation

between the plumb line and the frame. Only by Stage 3 is achieved the progressive independence of the various elements of the display.



Figure 4.5 Incorrect reproductions of the orientation of the plumb line

Piaget also observes that children's drawings of trees or chimneys suffer of the typical problem of lack of verticality.

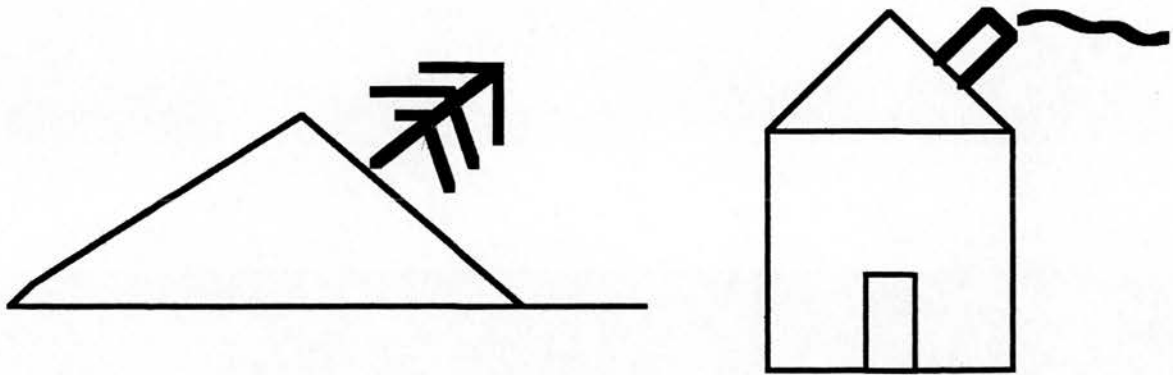


Figure 4.6 Children's incorrect drawings of trees on a mountain and chimneys on a roof

As is evident from these studies, the water-line task is set in a more complex framework. The results from the plumb-line problem or the chimney on the roof are directly comparable with the data from the water problem. The similarity in the type of errors children perform at all these tasks supports the view that there is a general problem in referring orientations to fixed abstract frame of reference. In all cases pre-operational children tend to use proximal elements as reference axes.

The acquisition of a system of coordinates is seen as the culmination of the psychological construction of euclidian space. It is the outcome of the coordination of the 8 infralogical operational groupings which are also responsible for the acquisition of other spatial concepts as the conservation of distances and the understanding of

topographical schemas. A frame of reference is, in fact, the product of logical multiplication applied to topological series which have been modified by the introduction of the concepts of straight lines, parallels, distances and angles, in N dimensions. This entails the employment of the whole set of euclidian concepts in order to link one object with another, thus constituting a global organisation of euclidian space which well surpasses the topological system of the previous stage.

A coordinate system or frame of reference presupposes, in the first instance, the topological notions of order and dimensionality. That is to say a set of relationships enabling objects to be ranged in series along 'n' dimensions. For example O-A1-B1-C1 along one dimension; O-A2-B2-C2 along another etc. But this is not all, for topological correspondence between pairs of the series takes no account of the distances between the members of the series. In contrast to this the correspondence between 2 or N series when they are incorporated within a system having 2 or N coordinate axes OA1B1C1 and OA2B2C2 conserve the distances $A1B1=A2B2$, $B1C1=B2C2$..etc. and also introduce a metric equivalence between successive intervals; viz. $O A1=A1B1=B1C1 =.. O A2=A2B2=B2C2=....etc.$ " (Piaget 1956 p.416 my translation)

Thus an essential property of the euclidian system, that only makes it accessible at the level of concrete operations, is the coordination of multiple elements. Piaget's theory of cognitive development relies on the idea that children acquire a greater mobility in the composition of elements and relations, in their mental combination and manipulation. The water level task and any other orientation task require such an operative mobility of the representational system, as they demand the coordination of the relative position of various objects. The parallelism between the infralogical operations and the logico-mathematical operations, e.g. assembling, relating and multiplying according to vicinity and similitude, or according to similitude and difference in the qualitative aspects in general, reflect the unity of function of operative reasoning.

4.4 Review of the replication studies of the water-line task.

The water-line task designed by Piaget to study the concept of horizontality has been replicated extensively. As for so many of Piaget's tasks, replications have confirmed the main effects described in the original experiment. However, there is also some evidence of contextual effect on the children's performance. A number of studies have attempted to control various performance factors, such as the role of drawing, the means of presentation, the difference between anticipation and copy, or even the physical knowledge involved in the Piagetian task. In general the question underlying

all these studies has been that of isolating the performance factors that may mask the real competence of the child. All the studies question Piaget's claim that children lack the vertical/horizontal reference system until the age of 10 or 11 years, and attempt to probe more systematically children's understanding of this notion.

I will examine most of the studies in the literature and give rather extended presentations of the procedures and results in order to highlight, in every case, the effect of the changes in procedure on the solutions of the tasks. The replications have been classified in three categories:

- 1) standard replications which have controlled for performance factors such as the means of presentation and of response or the difference between the anticipation and copy situation;
- 2) drawing studies which have examined the drawing requirements of the water-line task, and the drawing biases that children exhibit in their responses;
- 3) alternative studies of horizontality which have either modified the shape of the container or changed the physical setting of the task introducing a non-liquid version of the horizontality problem.

4.4.1 Replications that have probed performance factors

The replication researches which follow have tried to systematize and replicate Piaget's findings. Results confirm the essential conceptual nature of the task, as change in presentation medium or response medium do not significantly affect the results.

Smeslund (1963) was one of the first replications of the water-level task after the publication of Piaget & Inhelder's book. Smeslund's design was more systematic than Piaget's and consisted of pretest, observation, post-test. In the pretest the child was asked to draw the water level for bottles in different positions after having observed only a drawing of a bottle half full of liquid. Then a real bottle was presented and the experimenter tilted it to different positions asking the child to observe carefully. The child was then asked to draw the water-line on six pictures again. Finally the subject was asked to choose the correct pictures from a series of prepared cards.

Smeslund's results confirm Piaget's: there is evidence of a general failure to represent the horizontal among 5 to 7 year-old children. The data showed that in the pretest, 30% of the subjects had no correct drawing and less than 10% were correct in all six drawings. In the post-test there was a shift toward higher values, but the total

percentage of children with no correct drawings and with all correct drawings, remained unchanged. Subjects who did not produce any correct drawings in the pretest, did not profit at all of observation, but subjects who showed some initial trace of horizontality, i.e. those subjects producing 2 or more correct responses, improved somewhat in their performance (1 or 2 additional correct drawings). Pictures with the bottle at 90 and 180 degrees were found easier even more so in the post-test. The results in the situation involving the choice of cards were not significantly different from the situation involving drawing. Two tasks with different response means proved to be on average equally difficult.

Ford's (1970) study focused on the systematic comparison of performance in the anticipation and copy conditions of the water-level task. The study was conducted on 20 subjects from age 4 to age 6. The task did not require children to draw the water-line but to recognise the correct outline from a set of prepared drawings. Children had to choose one of four pictures representing the square bottle half filled with liquid tilted at various angles. All sets of cards consisted of one correct picture, one stage 1 picture (the water-level was represented parallel to the base for all tilts of the container), one stage 2 picture (water-level drawn diagonally from corner to corner of the bottle in accordance with Piaget's taxonomy) and one additional incorrect oblique picture (different from the other one). The subjects examined 16 sets of 4 cards. There were 8 positions of the bottle. The task was repeated once with the bottle hidden, in anticipation, once with the bottle visible, in copy.

No child showed full attainment of water line task in the anticipation (or prediction condition as it was defined by Ford), that is when the bottle was not visible. The number of correct responses ranged from one to six out of eight possible trials. In the copy condition (or perception as Ford defined it), where the bottle was left visible, performance improved. Correct responses ranged from 2 to 8 out of eight trials and 2 subjects had perfect performances. However 85% of the children had at least two errors.

% Of Tilt	0	180	90	30	45	60	135	150
Prediction	20	11	8	2	4	3	2	1
Perception	20	17	18	8	5	9	5	6

Table 4.1 Number of subjects producing correct choices for each jar tilt

Modal response on prediction task was the Stage 1 error in which the line is always parallel to the base of the bottle. This was particularly the case for the oblique tilts of the container. In other words in the anticipation (or prediction) condition, children chose more frequently the card on which the water was drawn parallel to the bottom of the bottle. In the copy condition (or perception condition) instead, the distribution was homogeneous for all 3 types of errors. Ford measured the consistency of the children's responses to determine whether children chose always the same type of card corresponding to the errors of a particular stage, or whether they modified their responses during the task. Out of 20, there were 6 non-consistent subjects for the prediction task and 13 for the copy task. The majority of consistent subjects gave responses of the Stage1 level.

In conclusion, results confirmed that 4/5 year-old children have a limited understanding of the water-line task, and that at that age in the anticipation situation there is a predominant tendency to do Stage 1 inversion errors. However in the situation where the bottle was always visible there are fewer Stage 1 errors and fewer subjects are consistently giving answers typical of a given stage. These differences were interpreted as suggesting that there are no grounds to defend a general stage analysis of the joint responses to the two tasks. The solution of the two tasks may not reflect a same unified general structure.

Both Smeslund's and Ford's study report that drawing and recognition tasks elicit the same effects. This suggests that the performance in the water-line task can be replicated with techniques other than drawing and that it is unlikely that the task is probing only a drawing skill. Both studies also give evidence that although errors can still be found when the subjects have the bottle to copy visible in front of them, this condition attenuates the errors when compared to the anticipation condition.

Thomas and Jamison's (1975) study which I will present next confirms this effect and gives further support to the view that the task is probing spatial and physical competence rather than simply drawing skills. Subjects in fact are asked to represent the orientation of the water-level with a three dimensional apparatus which is even further removed from graphical or pictorial representations.

Thomas & Jamison (1975) carried out a systematic assessment of Piaget's stages for the development of the notion of horizontality. The variables of age, sex, bottle shape and orientation are examined in detail. An interesting feature of this research is the experimental apparatus employed.

Subjects were asked to reproduce water level orientation on a 3 dimensional apparatus consisting of a cut bottle mounted on a vertical support. The section of the bottle could rotate on its support. A coloured cardboard disk was mounted behind the bottle to represent the water line and could be rotated independently from the bottle. On the side of the exercise bottle was mounted a real bottle half filled with water which could also be rotated. Two containers were used, a square bottle and a rounded flask. For each bottle the subject was asked to produce 24 settings, 2 for each position. The 12 positions corresponded to the hours of a clock.

The first situation was one of anticipation with the example bottle covered all through the experiment. The experiment involved 227 students from nursery to college. Results of study 1 show a decrease in mean setting errors with age. Three stages of development were identified as in Piaget's study:

- Nursery children made many inversion settings (line parallel to the base of the bottle). These errors corresponded to at least half of their answers;
- School children made fewer inversions, (significantly less than half of their responses) but aligned the water-line incorrectly. These stage 2 errors could be found over an age range that exceeded the 9/10 years limit described by Piaget;
- Finally stage 3 subjects made no mistakes at all.

Thomas & Jamison also found clear evidence that the performance in the orthogonal orientations is better than in the diagonal orientations of the bottle.

In study 2 with the bottle uncovered, the results were similar, except that the effect of all variables were sharply diminished and fewer inversion responses were found at all ages. Moreover a decrease in the angle of incorrect orientation appeared from grade 5 with 10 year-old subjects (the mean incorrect angle decreased from 30 to 10 degrees off the horizontal). In the covered situation the improved accuracy is spread over the period between 5th and 8th grade. In both conditions, anticipation and copy, the flask

was found easier than square and all the orthogonal positions were easier than the oblique.

An additional study with adults showed that success at the task was strongly correlated with the subject's capacity of explicitly stating the principle of invariance of the water level. In the anticipation condition 60% of adults did not know the principle and did average errors of 18 degrees.

The three dimensional apparatus employed by Thomas & Jamison provides further evidence that the errors in the water-level task are observable across a range of techniques and presentation modes. Not only are errors found regardless the representational means the subjects employ, but also, with different means, the same kind of errors are found at each age. In fact young children produce inversion settings similar to their drawings of the water-line parallel to the base of the bottle. Older children instead, produce 20 or 30 degrees settings more similar to their oblique drawings. This research also confirms the facilitatory effect of the flask over the square bottle, and the copy situation over the anticipation condition. Furthermore it confirms the earlier solution of the orthogonal orientations compared with the diagonal. These effects although not diverging radically with those presented by Piaget, have been more clearly isolated.

A very striking result of Thomas & Jamison's study is the significant number of adults doing stage two errors in the anticipation condition. A finding which has been frequently reported. In the sixties and seventies a few studies appeared that probed the adult's understanding of the water-line task. These researches were stimulated by the reports by Rebelsky (1964) who had observed 65% of errors in an adult population. These striking numbers were not confirmed by later studies Barna & O'Connell (1967) but it still appeared that a significant proportion of adults, maybe 20%, fail the task. Pascual-Leone (1970) related the phenomenon to field dependence and claimed that there were high correlations between succeeding the water-level problem and giving field independent responses to tasks such as Witkin's rod and frame test and other tasks measuring the cognitive style. In the same years it was suggested that mainly women adults fail; Willamsen & Reynolds (1973) correlated this result to women's low performance in field dependence/independence tasks, Thomas & Jamison (1975) relate it to an unspecified sex-linked factor. Angry replies by women researchers followed, as Musinger's article (1974). Overall, however, on either side none of these studies proved very conclusive.

Howard (1977) did the most recent study on adult's understanding of the horizontality of the water surface. The aim of the study was, in the terms of the

author, to investigate the perceptual recognition of and the explicit knowledge of the water-level principle in a life-like situation.

The apparatus was designed to maintain a natural setting: photographs were taken of a jug close to pouring liquid in a glass. The liquid level was made to appear tilted with respect to the glass and to the jug by tilting the background support and the glass itself. The scene was then filmed so as to make it look as though the room, support and glass were horizontal and the water line tilted. The photographs were either projected stereoscopically and gave a three dimensional impression to the viewer or were presented in a cinematic sequences. There was one correct sequence in which the background and support were left in their standard position and the water therefore appeared horizontal. The second sequence was the "trick" situation in which the water-levels appears tilted by tilting the background and table top. Subjects were shown the scenes and were asked to choose the most "natural looking" display, they were also asked to comment on their choice.

The results showed two distinct groups: one in which subjects perform with perfect precision and can state the horizontality principle explicitly (45% of the subjects), the second group accepted scenes with incorrect tilts and could not state the principle (55% of the subjects). Only the subjects that know a priori the fact of the constant horizontality of the water line can detect that in fact the only horizontal element in the display is the water level and that therefore the whole background scene is tilted. In order to understand the trick, subjects must choose the water as referent for all the other orientations. Subjects who did not know the principle were instead tricked into choosing the incorrect frame of reference while in fact the liquid in the jug functions as a spirit-level. Some of these subjects however performed correctly when asked to draw the water line. Howard observed that all the subjects who could express the invariance of the water-level performed perfectly on all trials, while the majority of subjects who did not express the invariance failed the task. At the same time, most of the incorrect subjects could correct their mistakes once the principle was explained to them.

With a different technique, Howard's data seems to replicate the findings of Thomas & Jamison, as both studies report 50% of adult subjects making errors in the task. I will come back to the import of this data for the interpretation of the development of the notion of horizontality . From now, we can conclude that the water-level task is more complex and abstract than Piaget had expected, involving explicit knowledge of physical principles.

4.4.2 Drawing biases

In the previous section, I have insisted on the broad span of techniques that have been employed to study the water-level problem. It has appeared that the typical errors are encountered with 3D apparatus, gesture, multiple choice and cinematographic methods as well as when drawing is used to represent the water-level. I consider that this finding constitutes sufficient evidence for the generality of the phenomenon to counteract the claims that Piaget's task is a drawing task probing a drawing competence. However since the task does a graphical representation of the water-line, an assessment of drawing strategies can be informative on the nature of the pictorial constraints children operate with and on the difficulties they may encounter. In recent years, in fact, there has been a shift from theories claiming that the child draws what he knows and that his incorrect drawings are an index of his incorrect representations, to theories attentive to the strategies and constraints of depiction. This led to the identification of a series of production biases. Such researchers in the development of graphical representation as Freeman (1980), would argue that these biases are sufficiently strong to override the conceptual intentions of the depicter. Ibbotson & Bryant (1976) have attempted to expose such drawing biases in the Piagetian water-level task and to explain errors, and in particular to stage 2 errors, where the water-line is drawn always parallel to the base of the bottle in terms of a specific perpendicular bias.

Ibbotson & Bryant (1976) explored the perceptual and pictorial structure of the water-level task and produced an explanation in terms of drawing bias which questions Piaget's account. Errors at the water-level task are in fact, interpreted as reflecting a failure in drawing non perpendicular lines. The authors argue that in both the horizontality and the verticality experiments used by Piaget & Inhelder all the difficult figures have in common a non perpendicular line which has to be copied and an oblique baseline from which the line must be drawn.

Ibbotson & Bryant suggest that the perpendicular error, i.e. drawing an acute or obtuse angle as a right angle, is a very basic geometrical error. Therefore the inversion errors that children perform at the water-level task can be seen as a manifestation of this drawing bias. The authors also report a vertical effect which suggests that children start by organising their spatial relations around the vertical axis.

Ibbotson & Bryant performed 3 experiments in which they controlled for the perpendicular error by asking children to reproduce lines coming from an oblique baseline and other non-perpendicular lines coming from horizontal and vertical

baselines. In the first experiment children aged 5 and 6 copied figures consisting of a long baseline from the middle of which protruded a shorter line, which was either perpendicular or oblique (45 degrees). The orientation of the baseline was also varied. Children had to copy the figure on a card on which the baseline had already been drawn.

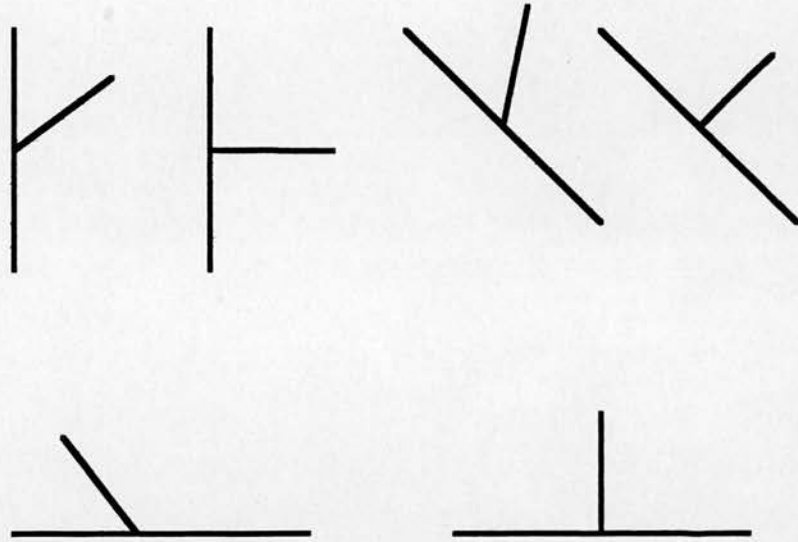


Figure 4.7 Some of the lines to be copied in the experiment of Ibbotson & Bryant

The results showed that 90° angles are copied more accurately than 45° angles. Moreover, the orientation of the baseline has an effect in the sense that the difference between results at the perpendicular and diagonal orientations are more marked when the baseline is horizontal or vertical than when it is oblique. There is a general tendency to perpendicularize lines: the lines tend to be drawn more perpendicular than what they should be, i.e. the incorrect angle tends to approximate the perpendicular even when the lines are drawn obliquely. This effect disappears however, when the baseline is vertical.

In the following experiment 3 and 4 year-old children were tested with the same 24 figures but had to reproduce the display not by drawing a line but by putting down a piece of wire. Results were similar to experiment 1, in the sense that 90° angles were found to be easier than 45° and the tendency to perpendicularize was confirmed, except with vertical baseline.

Finally in the third experiment 5 year-old subjects were asked to copy lines joining the two sides of a rectangle. Half of the lines were perpendicular to the sides and half were oblique. The rectangles were tilted at different angles. Ibbotson & Bryant compared the performance in this situation with the those in a meaningful situation in

which children had to draw the line within the outline of a mug tilted at different angles.

The results showed less errors with perpendicular than with the oblique lines and no difference between rectangles and mugs, in both situations the mean error for the oblique lines was of the order of 25 degrees. The perpendicular error was present in both cases.

Ibbotson & Bryant conclude that the presence of the perpendicular error with abstract material casts doubts on the hypothesis that the child does not understand the horizontality of the liquid. The child may have a clear grasp of the principle of horizontality but not be able to represent it correctly because of his general bias towards drawing perpendicular lines.

This explanation is reformulated by Bremner (1985) in terms which however, allow for some uncertainty.

The child may have been aware of the horizontal, but may not have produced it accurately as a result of powerful antagonistic effects.... Are local and global configurational biases sufficiently strong to produce error in tests of the horizontal and vertical despite a clear understanding of these planes on the part of the child, or do biases have an effect only when children's understanding or perception of the horizontal is still poor? At present we do not have sufficient data to answer this question. (1985 p.123)

In my opinion the data reported in Ibbotson & Bryant's study may shed some light on the nature of the inversion errors typical of stage 2. Ibbotson & Bryant's explanation of the perpendicular error however cannot account for all the behaviours observed in the different versions of the water-line task. It cannot for instance, explain children's stage 2B and stage 3A errors which typically are 20 or 30 degrees off the horizontal and not necessarily in the sense of more perpendicular to the side of the container, nor does it account for those drawings of lines which connect two corners of the bottle. In particular, the perpendicular bias explanation cannot account for the errors with round containers, where no perpendicularity can be established. Moreover Ibbotson & Bryant claim that the perpendicular error is a drawing bias and not a perceptual bias, because children discriminate very well right angles from non-right angles. Therefore the same "bias" explanation cannot be invoked to account for the errors with the 3D or the cinematographic apparatus nor the multiple choice technique.

Perner, Kohlmann and Wimmer 1984 confirmed Ibbotson & Bryant's results and at the same time showed that the perpendicular error was not encountered in selection tasks. The authors report that subjects as young as 3 years, given the choice between two drawings of a chimney: one correct and one perpendicular to the roof, were able to choose the correct one. Subjects recognized vertically drawn chimneys as correct when contrasted with perpendicularly drawn chimneys. Their own drawings of the chimneys (or trees or abstract figures) remained incorrect until the age of 7. However in the water-level task the difference between recognition and production did not appear as children were equally unsuccessful in either situation. Moreover when asked to point to the figure where the water was "straight", they judged the tilted level as the correct one in 80% of the cases. The authors argue that the problems with the liquid task lie in understanding the physical properties involved and that the incorrect depictions really do correspond to the children's conception of the orientation of liquids.

It is interesting to remark the Perner, Kohleman & Wimmer task differs from other selection tasks in that they gave the children a binary choice: correct or perpendicular. In Ford's task (section 4.4.1) for example, children had to choose from 4 pictures two of which had the water line tilted at angles of 20 or 30 degrees. In this condition children performed very similarly to their own productions and made many incorrect choices. Children's answers were equally distributed in the three types of incorrect depictions, thus reflecting more closely the patterns of their own drawings. The wider range of orientations of the water-line can better represent the solutions which the children find satisfactory. It may be argued that had Perner et al. used a more varied selection of drawings for the chimney task, they may have found more incorrect answers. In my opinion, their findings only support the conclusion that children disagree with the perpendicular representation and not that they accept only the correct one.

We can only conclude that, although it is plausible that drawing biases antagonize the horizontal the more the concept is unstable and poorly articulated, a unified explanation must be provided for the results in non drawing orientation tasks.

4.4.3 Replications that have modified the physical setting of the task

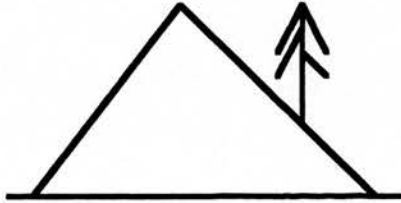
The two following studies are interesting in the sense that they have attempted to assess children's understanding of horizontality within different and more familiar situations than the water-level task. The first research requires subjects to represent the water level in a puddle. This task constitutes a minor modification of Piaget & Inhelder's task in that it only changes the shape of the container. The second study changes the task more radically and requires the subject to represent the orientation of a piece of meccano which can swivel on a pivot. This situation is strictly comparable to Piaget's task as far as its spatial requirements are concerned. On the other hand, it modifies the physical aspects of the display sufficiently to be an interesting test for the claim that children's failure at the water-level task can be explained by the complexity of the physical properties of liquids.

Mackay, Brazendale & Wilson (1972) discuss the issue of the *décalages* that have been reported between the vertical and horizontal tasks (Beard 1964). They claim that differences imputed to the rate of acquisition of the two concepts may in fact depend on properties of the tasks. In particular they suggest that the vertical task of drawing chimneys on the roof, is much simpler than the water-level task, a fact that may explain why it is solved before the horizontality task. The authors argue that that particular care should be taken in insuring that the tasks be of equivalent difficulty, as children of intermediate stages are very sensitive to methodological variations.

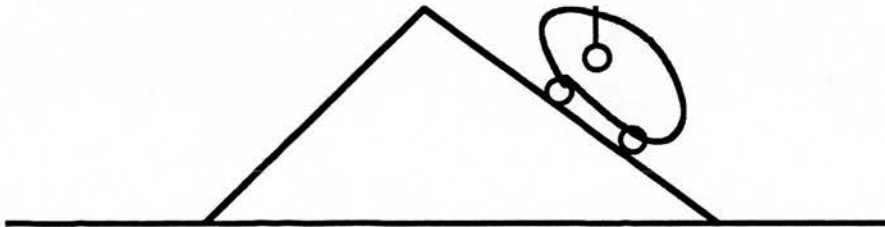
80 subjects aged from 6 to 9 years were presented with 4 tasks: the classic water-level task and an easier variation of it in which children must draw how the water is in a puddle outlined from a side view, the classic vertical task of drawing a tree on a mountain side and a more complex variation involving drawing a light bulb in a caravan driving up a mountain. The tasks were rated in difficulty by external judges, who evaluated the puddle situation easier than the classic water-level in the bottle task and the caravan situation more difficult than the tree on the mountain task.



Puddles.



Tree.



Light bulb in a caravan...

Figure 4.8 The tasks employed in the study of Mackay, Brazendale & Wilson

Results confirmed that at level 1, where horizontality and verticality are not yet present, and at level 3 where they have been acquired, there is no difference if a task is simpler or more complex. However the authors claim that at the intermediate stages:

..the decision whether a concept is available or not will depend to some extent on the form and content of the instrument used to investigate. (1972 p.236)

The simpler puddle task elicited a significant better performance than the standard water-level task. 75% of the subjects gave correct responses in the puddle condition against 1% in the classical bottle task. Within the age group examined, no significant improvement with age was found: 90% of 9 year-old subjects failed the bottle task as

did the 7 year-old subjects. In the puddle task 85% of the 9 year-old subjects succeeded the puddle task against 81% of the 7 year-old subjects.

	bottle	puddle		bottle	puddle		bottle	puddle
pass	0	17	pass	0	15	pass	1	30
fail	21	4	fail	25	10	fail	34	5
	Age 7			Age 8			Age 9	

Table 4.2 Number of children passing and failing the two tasks at each age

The predicted level of complexity of the task was confirmed by the order of acquisition of the vertical and horizontal. The simple version of the horizontal was acquired before the difficult version of the vertical and vice versa. The authors conclude that below the level of Piaget's substage IIIb the capacity to apply the concepts of horizontal and vertical varies depending on the particular situation. It is unfortunate that the authors do not advance an interpretation of what in the puddle display makes the task easier, and do not analyse what is difference between the two situations. It must be pointed out that both tasks were static and were both in anticipation, however in the case of the puddle, the hole is not supposed to undergo a rotation. The different orientations of the hole define different holes which each remain static.

DeLisi, DeLisi and Youniss (1977) discuss the complexity of the classic Piagetian horizontality task. They argue that the complex physical properties of the water are influencing the results in such a way that one cannot establish whether children's and adults' errors are due to a poor elaboration of the euclidian spatial system or a limited comprehension of the physics involved. A non-liquid assessment procedure was thus devised. The apparatus consisted of an object made of a base, a movable rod, and a crossbar. The rod could tilt with respect to the base, the crossbar however maintained itself invariably horizontal as it could swivel on a pivot joining it to the rod. The pivot was frictionless and thus the crossbar maintained a horizontal position for all the

orientations of the rod. The task therefore, had the same characteristics of the water-level task: both tasks involve an object which changes position but which preserves the horizontal orientation of one of its elements. In both tasks the child must draw a horizontal line. A comparison between the performance with the crossbar and water-level was carried out on 20 children from grades 1,3,5, and 20 college students.

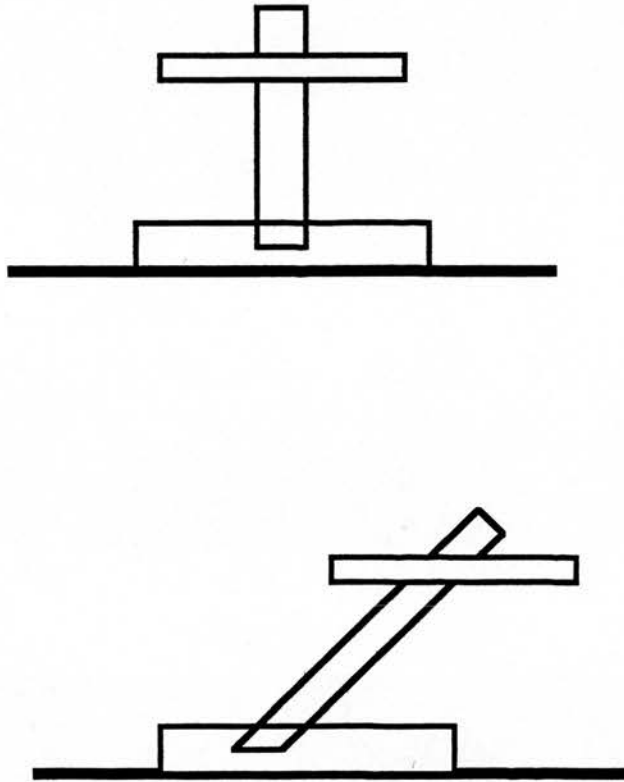


Figure 4.9 The crossbar apparatus in the upright position and the 45 degrees tilt

In the crossbar condition, subjects were presented a picture with the side outline of the base and rod. In the picture was missing the crossbar, children were hence asked to fill it in by drawing the crossbar. The apparatus was tilted at four angles 0° , 30° , 60° and 90° . For each tilt of the rod a new outline was presented and the child had to produce a new drawing. There was a preinspection phase with the crossbar hidden and an inspection phase with the apparatus visible. Finally there was a postinspection phase with the apparatus shielded again. The same procedure was used for the water-level task. The side outline this time was the bottle and subjects were asked to draw the missing water-level. A preinspection/inspection/postinspection design was also employed for the water-level condition.

Condition Grade	Bottle		Crossbar	
	Preinsp.	Postinsp.	Preinsp.	Postinsp.
1	.50	1.10	.35	1.60
3	1.40	1.50	.90	1.80
5	1.95	1.95	1.95	2.90
Col.	2.25	2.40	2.85	2.80

Table 4.3 Mean number of correct responses (scores ranged from 0 to 3)

Results confirmed the usual performance in the water-line task for 6 to 12 years old, youngest subjects failed both the oblique orientations as the 90° orientations. Older children succeed the 90° tilts but failed the oblique tilts. A similar pattern is replicated in the crossbar task. Hence, in both tasks orthogonal positions were solved earlier than the oblique orientations.

As for the comparison between the two tasks, performance did not differ for subjects of grade 1 and 3, where subjects failed the oblique orientations, however for grade 5, when subjects start drawing the oblique orientations correctly, and for adults, success at the crossbar exceeded success at the water-level task. However performance differences were observed only in the postinspection phase. In the crossbar in fact, the older children and adults were shown to benefit considerably from the inspection phase. The significant improvement after inspection, was obtained for the crossbar but not for the water-line task. This finding supports the authors' argument that the water line task requires knowledge beyond the system of euclidian coordinates.

Difficulties specific to the water task must be overcome to be able to demonstrate knowledge of the invariance of a liquid surface, but not to demonstrate its presumed basis in understanding of the horizontal coordinate. De Lisi et al. (1977 p.202)

Finally it is worth mentioning, that there have been a series of studies which simply employed the water-line task within larger empirical studies conducted to examine

other developmental paradigms such as learning or memory. In fact the relative stability of the behaviours in this task made it a good testing ground for the study of other psychological functions such as memory development or the effect of training on cognitive tasks. We can recall Liben (1974) on the relation of operative knowledge and long term memory, Liben & Golbeck (1980) on memory, DeLisi & Youniss (1976) in relation to mental imagery and Beilin, Kagan & Rabinowitz (1966) for training.

4.4.4 Summary of the data

In conclusion, the data I have examined give us the following picture of the development of horizontality:

- when asked to represent the water level in a container tilted at different angles, children draw it at an incorrect orientation, this behaviour which starts as soon as children represent the level with a line (around the age of 4 years), extends to the age of 11/12 years if not to adulthood;
- before the age of 4 years children draw the water as a mass. They draw little scribbles, or colour the whole area of the bottle. The conventional line representation is generally well established at the age of 5/6 years;
- the amplitude of the error (degrees off the horizontal) diminishes with age. Inversion errors, where the line is drawn parallel to the bottom of the bottle for all positions, disappear by the age of 7 or 8 years and are rarely found with adults;
- the first correct representations are produced when the container is at 0°, then correct responses extend to tilts of 180 degrees, then to the 90° tilts, and finally is generalised to all the oblique tilts,
- when the bottle and liquid are visible, it is easier for the children to minimize their errors (fewer inversions appear) or even to perform correctly. In this condition however, errors do not disappear completely;
- the shape of the container can facilitate the correct resolution: round flasks are found easier than square and "puddles" easier than bottles;
- errors are just as frequent in recognition tasks, when subjects are given prepared drawings, photographs or cinematographic sequences to choose from, as in drawing;
- there is some evidence that results in a non-liquid horizontal assessment are better than results at the water-level task;
- subjects who can verbally state the principle of invariance generally perform well in the water-level task with all means of response.

4.5 General Discussion

One of the first reactions Piaget's studies raise is usually one of disbelief. How can it be that children until the age of 10 lack the basic representation of horizontality and verticality? Therefore, some of the first questions these studies elicit, regard the particular setting of the experimental situation. The water-line task is no exception and the replication studies presented above have all investigated some of the performance factors that may be influencing the results and masking the child's real competence. Particular attention has been given to the type of response means children are asked to employ, as drawing is increasingly considered an unreliable means to assess children's spatial representations. Another common preoccupation that emerges from these studies is the complexity of the physical and dynamic properties of the situation: how much of children's failure in this task can be attributed to the lack of specific knowledge of the physics of liquids? Can the understanding of the physics be the real source of difficulty in the task rather than the geometrical knowledge? While these questions are present as an undercurrent in most of the studies, only DeLisi et al. challenge the problem directly by designing a task which manipulates the physical properties of the apparatus.

4.5.1 Are performance factors masking the real competence of the child?

The first overall remark that can be drawn from the previous review is that the phenomenon exposed by Piaget in his study of the horizontality of liquids is a relatively solid finding. Although only a few variables have been examined, Piaget's analysis has not yet found any serious antagonist. One conclusion that can in fact, be drawn is that there is sufficient evidence to consider that modifications in performance factors as the kind of display and the medium of response, do not significantly affect the results. The wide range of techniques employed to test the water-line phenomena have confirmed the conceptual nature of the problem under scrutiny. The different techniques used in these studies, be it the 3D apparatus, the cinematographic sequences or simply the prepared cards, have confirmed that children's errors are not simply drawing errors. Results in all these situations are all comparable to the original results found by Piaget & Inhelder. It appears therefore that their task does not embed particular performance requirements that mask the real competence of the child with the exception of subjects younger than 4 or 5, who are unable of representing the water

level as a line and tend to scribble. A closer investigation of the spatial competence of this age group would certainly require different task settings in which drawing should not be involved.

A significant difference in response has only been found between tasks where the subject was asked to anticipate the orientation of the level and tasks where the bottle was visible at all times. Not only the copy situation elicited more correct responses, but some characteristic errors as the inversion error (line parallel to the bottom of the bottle) were significantly less frequent than in the anticipation. These results are suggestive as to the different nature of the two experimental conditions: the anticipation task requires subjects to know the physical phenomena of the invariance of the water-line a priori, the copy task is an orientation task which requires subjects to represent the spatial relations between the elements of the display. In the copy situation, theoretically, the prior knowledge of the invariance principle is not necessary but can be discovered there and then. In fact, it is not even necessary to abstract a general principle and each orientation can be treated independently as a new problem of relating one plane to the other elements via their mutual relation with the external frame of reference. This distinction should be kept in mind in designing experiments probing this notion and a clear choice should be made as to exactly what type of knowledge is to be assessed. Despite these differences, both task conditions elicit *some* errors in young children and there is no clear disappearance of the phenomena described by Piaget, in the copy situation. In conclusion the errors found in the horizontality problem cannot be simply imputed to a performance problem and once we exclude the role of performance factors we are led to conclude that the change in response children exhibit at different ages, reflects a genuine change in competence.

4.5.2 Is the water-line task in fact assessing knowledge of physics?

The second source of possible error are the physical properties of the objects involved in the task. The hypothesis may be made that while Piaget thought he was investigating the capacity of representing horizontal planes, the task is in fact probing the development of knowledge of the physics of liquids. DeLisi, DeLisi & Youniss approach the problem from this perspective and provide interesting data that partially supports this view. Their task has kept the structural characteristics of the water-line situation but has embedded the situation in a different physical context. As in Piaget & Inhelder's task there is an apparatus composed of two elements which are dynamically independent although one is the support of the other. Therefore as in the water-line

task, a change of orientation of one element entails a change in the spatial relation between the two elements. However no change occurs in the relation between the crossbar and the environment as the crossbar stays invariably horizontal. The physical, dynamic reasons of the invariance of the crossbar have to do with the properties of the pivot (the fact that it is frictionless and that equal torque is applied to both sides of the balanced bar) and are certainly different from the reasons that maintain the water surface horizontal.

In this situation, DeLisi et al. found a slight anticipation in the correct responses with regards to Piaget's task. The anticipation regards essentially children of age 11 (class 5). The crossbar apparatus also augmented the number of correct responses in adults. The interesting fact is that the improvement in number of correct responses was found essentially in the postinspection phase. There was in fact no significant difference in the tasks in the preinspection phase, but all children showed significant improvement due to inspection (with 5th graders performing no differently than adults). This suggests, in my opinion, that subjects did not have an a priori understanding of the dynamics of the apparatus but were capable of understanding and representing the spatial relation holding between the bar and the pivot when they observed the spatial transformations. It is the case in fact, that the physical reasons that determine the invariant orientation of the crossbar, are arbitrary and the apparatus has been manufactured in such a way to maintain the bar always horizontal regardless the orientation of the support. It is difficult to imagine subjects anticipating the working of this little system.

This fact suggests two remarks, the a priori knowledge of the dynamic independence of the water from the container is as complicated for children as that of an arbitrary apparatus. After observation however all the children and older subjects especially, can more easily represent the orientation of the crossbar than the water. The discovery of the invariance of the crossbar is easier than that of the water.

When we relate this data with the evidence from the studies on adults which reported a considerable number of failures in the water-level task, the conclusion that one is invited to draw is that the classical Piagetian task is an extremely complex task involving various domains of knowledge. Although the task of representing the orientation of the liquid surface has become a conventionally accepted means of assessing knowledge of horizontality, other situations should be envisaged to understand the development of this notion. In particular, I would argue that if one of the problems subjects are faced with is that of realising the dynamic independence of the liquid surface from the bottle (and that changing the orientation of the container transforms the relation of the water and the bottle), different physical apparatus should

be employed. Apparata for which subjects may have a clearer understanding of the dynamics of the objects involved. I would also suggest that, contrary to the crossbar paradigm, the apparatus used should not be constructed arbitrarily as to maintain a plane horizontal but should involve objects of which subjects can predict the orientation relying on their knowledge of physics⁶.

In conclusion although DeLisi, DeLisi & Youniss have brought suggestive evidence on the physical knowledge required in Piaget's water-level task, their data on children's errors at the crossbar task give some indirect support to the spatial interpretation as well. Children before grade 5 perform badly in both tasks and their errors are concentrated around the oblique positions of the crossbar rather than the 90 degrees inclination, just as is the case with the bottle. In other words children's difficulty in orienting the plane of the crossbar is somewhat comparable to their difficulty in orienting the water-level. The question that arises at this point, is the following: if children have a real problem (and the review has definitely confirmed that it is a competence problem) in representing the horizontal orientation of some specific plane, be it the water-level or the crossbar, what are the reasons of this failure and what is the developmental process that leads the child to the correct representation? Once we have eliminated all the hypotheses of performance factors masking subjects' competence, what are the reasons for children's misrepresentations in these tasks? More specifically, the question is whether we are going to subscribe to Piaget's explanation of this phenomenon and whether this explanation is fully satisfactory? In the following paragraphs I will argue that Piaget's analysis of the horizontality phenomenon is insufficient to explain all the data and is incorrect in some of the interpretations.

4.5.3 Piaget's theory of spatial coordinates, is it a good explanation?

Let us reconsider Piaget's task from a different perspective and reinterpret the task requirements. The task of the water-line involves three objects: a container (the bottle), a quantity of liquid of which the only relevant part is its surface, and the environment of which the table top is a token element. There are two objects which are mobile, the

⁶ As far as a change of type of container is concerned, there is the case of the puddles of Mackay's study. The anticipation effect obtained with the puddles can be attributed more to the fact the situation is static, than to the actual shape of the bottle. This experiment has changed radically the requirements of the Piagetian task in that no visible transformation occurs in the relation between container and liquid. However the lack of acute angles can also be playing a role in the simplification of the situation.

bottle and the water. These two objects stand in a specific spatial relation. If we take the canonical position of the bottle sitting upright on the table to be the starting situation, this relation corresponds to one of perpendicularity. The surface of the liquid is perpendicular to the sides of the bottle. The task can be seen as one of transforming this spatial relation. A transformation of the position of the bottle, which modifies its spatial relation with the environment entails a transformation in the spatial relation holding between the liquid surface and the container. From the physical point of view therefore, the two objects (bottle and water) are dynamically independent.

To recapitulate, there is a transformation in the spatial relation between the bottle and the table which entails a transformation in the spatial relation of the bottle with the liquid surface (the perpendicularity is broken). However the spatial relation between the water-line and the environment stays unchanged. Because of the physical properties of liquids the surface of the water stays invariably horizontal whatever the inclination of its container.

If we look at children's incorrect responses, it would seem that initially children maintain the same spatial relation between the surface and the bottle, they maintain the perpendicularity and therefore draw the water as if it was stuck to the bottom (stage IIa). At a subsequent stage children seem to realise that the initial relation is broken and they attempt to establish a new relation which takes into account the dynamic independence of the liquid. Their drawings at this stage avoid the perpendicular but are at incorrect angle of tilt with respect to the horizontal. At stage IIIa the correct relation between water and bottle is re-established but only for the orthogonal positions of the container. Finally the correct relation between water-level and environment is determined for all positions.

The developmental question seems to be the following: why do children maintain the relation between the surface and bottle invariant rather than maintaining the relation between the surface and the environment which truly stays invariant? For Piaget the answer is clear, children of the concrete operational stage can only represent proximal relations. Therefore they cannot relate the plane constituted by the water-line with a distant element of the environment. Furthermore they lack the geometrical apparatus of euclidian geometry (e.g. right angles, parallelism, distances etc.) that would allow them to establish the correct spatial relations between the various elements of the display. In short for Piaget, children at this stage simply lack the general operational means to represent the spatial relations that hold between the water and the environment. But is this really so? Let us examine Piaget's argument more closely.

There are two relatively distinct levels of explanation Piaget employs to explain children's errors. A first level which we may call functional, concentrates on the lack

of the representational mobility that allows a subject to relate multiple spatial entities, progressively more distant within an organised system of relations. A second level of explanation is concerned with a more structural definition of the type of spatial organisation that characterises each stage; a topological geometry versus an euclidian geometry. According to this interpretation children who represent space in topological terms do not have a fixed system of horizontal and vertical reference to which refer all the orientations of objects and therefore fail to recognize the invariance of the water-line with respect to the environment. Neither level of explanation seems free from some problem. I will discuss the two explanations in turn.

4.5.3.1 The transition from proximal to distant spatial relations.

According to Piaget's definition of an operational use of cartesian coordinates, children that fail the horizontality task by incorrectly orienting their drawing of the water surface, lack the representational power to coordinate mobile and immobile entities within one organised system of relations. The most crucial acquisition of Stage IIIb, is to be able to coordinate indefinitely more and more elements within a progressively larger system of spatial transformations.

Piaget claims that the transition from failure to success in the water-level task can be seen as a gradual process from a system that privileges proximal relations between objects to one that is independent from any specific object and can therefore relate all objects, their positions and the distances between them. Similarly there is a progressive extension of the reference systems, which move from proximal to distant. In other words Piaget considers that there are proximal entities taken as references, that anchor spatial relations before the abstract system of coordinates is in place, however these proximal references do not have the property of invariance that allows all the mobile objects of the display to be coordinated with regards to an independent frame. The reference of the sides of the bottle is incorrect as it is an element that is itself involved in the spatial transformation of the display. The children's choice of this mobile and therefore incorrect referent, is to be imputed to their incapacity to represent all of the transformations in the display and thus identify that the sides of the container are themselves mobile elements.

I see two problems with this argument. The first regards the data found with adult subjects. It is difficult to attribute the same type of difficulty in relating distant elements or coordinating multiple relations between objects, to the considerable number of adults failing the task. Moreover there are grounds to consider that the problem the task poses is not really one of relating multiple positions and orientations.

In the classical setting of the task, the operational child can rely on the frame given by the table top, draw the water line parallel to the table and therefore orient the water line correctly in a horizontal position. In this case the child behaves as Piaget would expect from an operational subject, in that he has chosen a referent external to the two mobile elements of the display, the water and the bottle. However we may think of mischievous experiment in which the table itself was tilted of 45 degrees. In this situation, establishing a parallelism between the table top and the water line would lead to an incorrect answer. In this case as in the previous one, the subject would have shown the capacity to coordinate a number of elements of the display and coordinate their spatial relations. The problem the trick situation highlights is that of choosing the *correct* reference. To give a correct answer the child should have chosen as referent the floor of the room. But if one wanted to be grandiose, the whole room could be tilted, and again the reference of the floor would be incorrect. Apparently then the question is not really that of spatially coordinating and relating multiple elements, orientations or positions, but rather of identifying the relevant frame of reference for the situation.

This raises a further question: what is the distant element that has to be taken as referent? It is with what element of the environment that the spatial relation has stayed invariant? Following Piaget we assumed it was the table because it was the next in order of proximity and because it serves as support to the bottle. I would argue, on the contrary, that the only secure criterion to determine the horizontality of a plane is its perpendicularity to gravity. Ultimately the correct plane in the environment to be chosen as referent for parallelism, is the plane that is perpendicular to the direction of gravity. In order to determine that the table top or the floor in our hypothetical mischievous experiments are good or bad references for the orientation of the water line, one has to establish whether they are themselves horizontal. To do so in absence of any further parallel referent requires to establish if these planes are perpendicular to gravity or not.

Going back to our initial definition of horizontality, a physical object is considered horizontal when it lies on a plane perpendicular to the direction of gravity and parallel to the line of the horizon. It seems that in absence of frequent exposure to environments where the line of the horizon is visible, the direction of gravity constitutes an essential property of our physical world on which horizontal and vertical planes are constructed. Gravity appears to be the primary cue to the assignment of verticality and horizontality a point on which there is certain agreement, (e.g. Olson & Bialystock 1983 p.76). In contrast to Piaget's interpretation I would therefore argue that the task children are set to solve is not to establish the parallelism of the water-level

with the table top or with the floor, but the task is to establish that the water-line is perpendicular to the direction of gravity. It may seem a subtle difference but I think this point has wide ranging implications in explaining children's errors in this task. The spatial relation to be discovered is not therefore, between the water-line and some element more or less distant of the environment, the relation must be established with a specific plane or direction, that of gravity. The issue of being able to coordinate relations of objects which are progressively more distant, is transformed into the problem of establishing the perpendicularity of a plane to the direction of gravity.

4.5.3.2 The transition from topological to euclidian geometry

The second level of Piaget's explanation regards the transition from a topological to an euclidian representation of space. Piaget considers that young children initially represent space solely in topological terms and the only spatial relations understood are the topological notions of proximity, neighbourhood, enclosure etc. These are the sort of relations that young children represent by the little scribbles they draw within the boundaries of the bottle. At the end of the developmental process, children on the contrary, represent space according to euclidian properties. In this system the child can not only represent relations such as perpendicularity, parallelism, oblique angles, etc., but space is organised as an empty container in which all relations can be determined in reference to a stable system of coordinates. Piaget makes no proposal to characterise the specific spatial theory held by subjects in the intermediate stages.

In my opinion, the productions of the intermediate stages cannot be simply characterised in terms of topological relations because the utilisation of lines and of spatial relations such as parallelism and perpendicularity (as is evident from the correct answers with the round container or when the bottle is tilted at 90 degrees and even when the parallelism is established with the bottom of the bottle), go beyond such system and are an index of the understanding of some euclidian properties. If we look at the type of incorrect drawings of Stage IIa, IIb, IIIa, they all consist in drawings of lines which are either kept parallel to the bottom of the bottle or are incorrectly oriented at an angle greater than 0 degrees. In the case of Stage IIa drawings in which parallelism is established with the bottom of the bottle for all positions, subjects are undoubtedly using a property of euclidian geometry, namely parallelism.

Similarly subjects who are correct in their orientation of the water-line for the 90 degrees orientation of the bottle while still failing the other positions (Stage IIIa), are also capable of employing parallel and perpendicular relations. These subjects in

particular pose a real problem to Piaget's interpretation. Before succeeding for all the orientations of the bottle children are capable of representing the correct orientation of the water-line when the bottle is at 90 or 180 degrees. Although it is arguable (and I will be arguing precisely this point in the next chapter) that these conditions are spatially simpler than the situations in which the bottle is tilted diagonally, it is undeniable that children are here employing euclidian type relations and are representing the water surface horizontally (via which reference frame is an open question). It seems hence incorrect, to deny that these subjects possess some of the elements of euclidian geometry that Piaget is willing to attribute only to the subjects of Stage IIIb. The question which arises at this point, is whether the "intermediate" child in view of the fact that he does not solve the task in all its conditions, can be attributed with the same type of euclidian system as the older child or adult.

Unfortunately Piaget does not provide any clue to solve this problem as he focuses only on the extremes of the developmental scale: the initial topological system and the final euclidian one. For the long intermediate stage going from the phase in which the water is drawn as a scribble to the stage in which all orientations are correctly depicted, no specific analysis of the type of spatial relations used at this stage, is provided. The various intermediate behaviours are seen as mere steps from the topological system to the euclidian. In fact for Piaget, Stage IIIa drawings of a horizontal water-line in the 90 degrees condition are seen as a first step towards discovering the invariant orientation of the water. Via the coincidence of the side of the bottle and the table top, the child can use the parallelism with an internal element of the bottle to start conceiving of the relation that holds between the water and the external framework of the display. Therefore, for Piaget these results are taken as evidence of the progressive externalisation of the reference for horizontality, while I would argue that they bring fundamental support to the view that children have constructed the basic euclidian relations well before they can succeed the water-line task as a whole and demonstrate that it is not by lack of such notions as parallelism or perpendicularity that children fail the task.

If we were to go along with Piaget's hypothesis that children's failure to orient the water level is an index of their general inability to organise spatial relations around a system of vertical and horizontal coordinates, a specific proposal should be made as to the nature of the geometry or spatial system that intermediate subjects work with. Neither the topological system nor the euclidian system as defined by Piaget, seem to capture the particular nature of children's spatial representations during the period covering substages IIa, IIb, IIIa. These substages represent at least 5 years of development as subjects' ages in this phase range from 5 to 10 or 11 years. It consists

therefore of an extremely significant portion of the population studied by Piaget and Inhelder and constitutes a very long preparatory phase before succeeding at all the conditions of the task. There is an abrupt transition in the theoretical explanation from the topological system to the euclidian system, as Piaget makes no attempt to capture the consistency underlying the responses of the intermediate period⁷. It is a legitimate question to ask whether we can define a coherent spatial organisation for this period of development. A spatial system that underlies the diverse responses of the intermediate stages and which therefore, embeds some of the properties of the euclidian system while still not embedding some of the features that allow older children and adults (at least the successful ones) to correctly solve the water-level task.

4.5.4 Conclusions

In conclusion, Piaget's explanation of children's errors at the water-line task relies on two related arguments: children's lack of the euclidian geometrical conceptual apparatus, and their inability to relate multiple elements of the display to an immobile referent. In the previous paragraphs I have argued that neither of these factors seem to explain satisfactorily subjects' performance. The capacity of subjects to draw lines parallel to internal elements of the bottle show that they are using properties of euclidian geometry to represent the water level. The issue of having the representational power to relate distant elements of the display such as the water and the table top, seems to be inadequate to explain the errors of adult subjects in the task. Furthermore I have attempted to show that the critical problem of reference is not that of relating the water surface to a stable support as the table, but that of identifying that the water-level is perpendicular to gravity and that any plane chosen as referent for horizontality must be chosen on the grounds that it is perpendicular to gravity.

If we exclude the explanation of children's failure at the water-level task as the lack of the representational capacity to relate the water-line with an external referent for horizontality (as the table top), it remains to be explained why subjects consistently are

⁷ In fact I think that the general reasons for which Piaget does not attempt to give a precise structural characterisation of the intermediate stages, are probably to be imputed to his rigid reliance on Klein's (1939) hierarchy of geometries as much as on his theory of general stages of cognitive development. On the one hand, euclidian geometry is considered the most restrictive member of Klein's hierarchy (the fewer transformations admitted in this system make it the more powerful) and thus Piaget considered that it could only constitute the end-point of spatial development. Piaget does not in fact seem to have at his disposal, or to be willing to define, a formal geometry that takes into account the specific spatial relations established by his intermediate subjects

orienting the surface of liquids to some element internal to the bottle. It is very clear from the evidence at our disposal that subjects attempt to refer the water-level to the internal frame of the bottle rather than to a fixed horizontal plane. The developmental question remains, as I have argued previously, why children concentrate solely on the relations holding between the water and the bottle and ignore the relation holding between the water and the environment.

It is also not clear, whether the reliance of these subjects on the internal frame of the bottle can be characterised as a general feature of their spatial representation. The fact that subjects are using relations such as perpendicularity and parallelism, relations that belong to euclidian geometry, means that their spatial representations cannot be defined solely in terms of topological relations. But how is one to capture the particular elaboration of these relations within the specific space of the bottle? What type of spatial organisation characterises children that seem to be incapable of determining the orientations of some elements of the display while being capable of employing some of the conceptual apparatus of euclidian geometry to define the relations holding within the object? Can we characterise these responses as the expression of a coherent and organised spatial system that precedes the euclidian system as defined by Piaget for the final stage in the water-line problem? The most pressing question becomes therefore, to examine more closely the type of situations in which "intermediate" subjects are successfully employing euclidian relations and thereby characterise the extension and limitations of the spatial notions they are operating with during this phase. In particular special attention should be brought on the situations in which children seem to succeed more easily in reproducing the correct orientation, namely the orthogonal positions of the bottle (90° , 180°) and the round container.

The next chapter will introduce an empirical research that has focused on the issue of defining the competence of intermediate subjects. By decomposing the water-level task in a series of conditions which control for such factors as the angle of tilt of the bottle and the shape of the container, I wish to provide a finer grain analysis of the type of relations children of the intermediate stages seem to be establishing between the water-line and the elements of the bottle.

Chapter 5: Experiment 1

5.1 Introduction

In previous chapters I had suggested that a fruitful way of approaching conceptual development, was to concentrate on the *transformation* of the child's concept rather than attempting to establish when the full blown concept appeared. The analysis of Piaget's theory of the development of horizontality has shown some of the limitations of the latter approach, as it fails to provide a satisfactory account of the competence underlying the first correct representations of the water-line when for instance, the bottle is tilted at an orthogonal angle and when the container is a round flask. I wish to approach the analysis of these first forms of representations of horizontality from a different perspective, which consists in assessing whether there is a competence level of some coherence and complexity prior to the "full blown" notion of horizontality of the last stage. The analysis of Piaget's data and the replication studies suggest that there are the grounds to think that children of "intermediate" stages have some of the spatial competence that Piaget attributed only at the last stage of the acquisition of the water-level problem.

The previous review suggested that the process leading from incorrect to correct representations of the water-level is not abrupt, that subjects do not pass directly from failing all conditions to succeeding them all. It has emerged that there are intermediate phases in which subjects are partially correct, in that they are able to represent the water-line horizontally for some angles of tilt of the bottle and with round containers before they succeed in all conditions. Piaget reported an order in the resolution of the task that went from the initial correct solution to the orthogonal tilts of the container to the later solution of the diagonal orientations of the bottle. Ibbotson & Bryant, Ford, De Lisi, DeLisi & Youniss, Thomas & Jamison, also reported that children first succeed in reproducing the horizontal position of the surface of liquids when the bottle is upright, then when it is at a right angle and only subsequently when it is tilted at 45 or 135 degrees. De Lisi, DeLisi & Youniss described a similar effect with the crossbar, where 0 and 90 degrees inclinations of the crossbar are found easier than 45 degrees tilts.

What do these results indicate on the nature of children's understanding of the spatial relations involved in the situation and what do they tell us on the type of spatial organisation of this period of development? The capacity to establish relations of parallelism and perpendicularity, the use of straight lines and angles when the bottle is tilted at 90 or 180 degrees, or when the container is round rather than square, shows

that at this level, the child has many of the ingredients to represent correctly the orientation of the water level even though he fails the diagonal conditions of the task. It seems therefore useful to examine systematically the extent of these children's competence and failures, in order to determine whether the responses typical of the long intermediate stages have a global underlying coherence which can define a consistent spatial organisation preceding the euclidian system defined by Piaget only for the final stage of the water-level task.

Piaget designed the task as a whole and true understanding of the principle of horizontality was attributed only to subjects capable of anticipating the correct horizontal orientation of the water-line for all 8 angles of tilt. However in my opinion there are grounds to consider that the orthogonal conditions have different spatial requirements than the diagonal tilts of the bottle. As was mentioned in the discussion of the previous chapter, the task of the horizontal involves a transformation of the spatial relation holding between the surface of the liquid and the sides of the bottle. Consider the relations involved in the diagonal orientations of the square bottle: from an initial canonical relation of perpendicularity between the water-level and the sides of the bottle and of parallelism with the bottom and top, a tilt of the bottle of 45 degrees brings about a new relation between water-level and the sides of the bottle. The water-level is now oblique to the sides and not parallel to the bottom of the bottle.

If we consider the 4 orthogonal positions 0° , 90° , 180° , 270° , there is no transformation in the type of relation holding between water and sides, in every case perpendicularity is established again even though this may be with different elements of the bottle. When the bottle is tilted at 90 degrees the water line is perpendicular to the flat bottom and top of the bottle rather than to its sides. Furthermore, when the bottle has orthogonal positions, the same relation holds between the water-line and the bottle as between the water-line and the direction of gravity: perpendicularity again. In other words, the axes of the external coordinate system of reference and the axes of the bottle coincide. Thus, orthogonal positions of the bottle do not involve the same transformation in the relations between the various elements of the display, as do the diagonal tilts of the bottle. While in the diagonal positions of the bottle it is necessary to consider the relation between the water-line and an external referent (be it parallelism with the table or perpendicularity with the direction of gravity) and only the external referent can enable the subject to establish the correct orientation, in the orthogonal positions both internal and external references are reliable and relevant referents.

With regards to the round containers, the same argument can be made as for the orthogonal orientations of the square bottle: the change in orientation of the flask does not bring about a change in the relation between the water-line and the container. The

rounded shape of the flask with its symmetry and absence of angles, maintains invariant the angular relation of the level with the internal rounded sides of the flask. Therefore this situation is again simplified with respect to the diagonal positions of the square bottle. The child does not really have to look for an external frame of reference but maintains the same spatial relation between bottle and liquid for all its orientations.

A very tentative account of the type of spatial organisation that may be underlying the correct responses in these conditions may go as follows: success in these conditions and the concomitant errors in the diagonal positions of the square bottle, may be the index of a spatial organisation that may be defined as object centered. Children may be capable of employing the relations of parallelism, perpendicularity and even of horizontality only within the space of the bottle. The container functions as a frame and provides horizontal and vertical axes to which the orientation of the water can be related. When the frame offered by the container does not coincide with the axes of the environment the child is at loss to define the orientation of the liquid.

However before proposing any type of unitary interpretation to account for children's behaviours during this period it is necessary to establish whether there is a consistent concurrence in the solution to the orthogonal conditions that would allow us to envisage a coherent organisation of their representations, and whether the responses support the view that there is a unitary stage in the development of horizontality that could be described as "object centered"⁸. The experiment which follows has been designed to assess the orders of acquisition between different conditions of the water-level task, and in particular to establish whether there is a relation of concurrence in the solution amongst the orthogonal positions of the square bottle and between these positions and the round container.

5.2 Objectives

The main objective of this research is to establish a developmental hierarchy between conditions of the water-line task and to identify stable stages in the development of the notion. In particular I will investigate whether orthogonal orientations of the bottle

⁸ None of the studies mentioned above in fact, has examined systematically the developmental order holding between these different conditions. It is not clear for instance, whether the four orthogonal positions are solved simultaneously and whether they all systematically precede correct representations at the diagonal positions. Similarly it is not clear whether success at the round container precedes or is concurrent with success at the orthogonal positions of the square container. No specific order was established between the the round container and the different positions of the square one. It was simply suggested that the round container had configurational properties that more easily allowed the child to look for an external parallel referent.

elicit systematically correct answers before diagonal orientations, and whether the relation of precedence between the round and square containers is a constant or is specific to some orientations. In fact, if there is a competence level of some coherence and generality before the "full horizontal" of the last stage, then we should expect that the four orthogonal orientations of the square bottle be solved concurrently. At the same time we would expect that round containers elicit successful responses concurrently with the correct reproductions of the orthogonal positions of the square bottle. On the contrary a collective *décalage* should hold between these conditions and the diagonal orientations of the square bottle, orientations that according to our hypothesis are introducing a different type of spatial relation.

In order to examine the effect of shape on children's responses, I have substituted the round flask generally employed in studies of the water-level task, with a glass sphere. Until now the only round bottles employed in the water-level experiments have been laboratory flasks which, although have a rounded bottom, also have a long straight neck which could have been used as reference for parallelism or perpendicularity. In my experiment, I will be employing a completely round glass sphere which simply has a little spout, more like a bump, closed by a tiny cork, which functions as an indicator of rotation. Since a sphere and its bidimensional representation, the circle, are symmetrical, a change in orientation does not modify perceptually the relation between its parts and the environment. It is necessary therefore, to have an marker on the sphere that changes position when a rotation is imposed on the sphere. Contrary to the flask used by Piaget & Inhelder, the sphere has no straight element that can come to create a right angle or a parallel with the water-line. Hence, in this situation the transformation in the relation between the sphere and the environment e.g. the rotation, does not involve a change in the relation between the water-line and the container. Not only does the water-line stay invariably horizontal with regards to the environment, but there is no visible change in its relation to the sides of the sphere (if one excludes the position of the spout). We are therefore in a similar situation to that of the orthogonal positions of the square bottle where again there is no real transformation in the spatial relation between the water line and the container. Furthermore the absence of significant geometrical differences between the various orientations of the round container would predict an annulment of the orthogonal effect; no difference should be found for the different tilts. Overall the glass sphere seems to isolate more clearly than the flask those configurational properties that could explain the improved performance with regards to the square bottle.

In sum, while replicating the water-level task, the objective of this first experiment is to investigate systematically the role of two variables: the shape and the angle of tilt of

the container. By establishing whether a systematic concurrence can be found in the correct solution of the orientation of the water-line for certain conditions of the task, a fundamental step can be made in assessing the coherence of children's representations in the period preceding the correct solution to all conditions of the water-level task.

In order to analyse the order of acquisition of the different task conditions, a number of preliminary questions as to the relation of the different variables must be answered. The first question to be addressed is whether there are grounds to classify the different orientations classically used in the water-level task, in two categories: orthogonal and diagonal. Whether, in other words there are significant differences in the responses to the different angles of tilt of the bottle, and if these differences discriminate between the categories and not within. Similarly it is necessary to examine the performance differences for the two shapes, round and square, and the relationship between shape and angle. In other words I wish to establish whether it is possible to distinguish 4 distinct task conditions: diagonal tilts of the square container, orthogonal tilts of the square container, diagonal tilts of the round container, orthogonal tilts of the round container.

At a first level of analysis we will attempt to answer three questions:

- if there is a significant difference between the angles of tilt and if this difference is equivalent for both shapes of containers,
- if there is a significant difference between round and square containers
- if there is a significant difference amongst the responses of children of different ages and if this difference is more marked for some shapes and orientations

The subsequent level of analysis will regard the question of the hierarchy of solution of the different modifications of the task. If the results in the first level analysis will allow us to discriminate between the diagonal and the orthogonal orientations, and between round and square bottles, we will be considering each category as a different task. There will be therefore 4 tasks: orthogonal square, orthogonal round, diagonal square, diagonal round. An order of solution will then be established between the 4 tasks, by examining the response patterns of correct and failed for each pair of tasks (e.g. collective *décalage*, concurrence). The aim of this second analysis will be to determine whether a clear hierarchy in the solution to the 4 tasks can be established.

5.3 Hypotheses

The first hypothesis regards the difference between the two categories of angles of tilt in the square bottle, a significant difference in response is expected between diagonal orientations (45°, 135°, 225°, 315° angles) and the orthogonal orientations (0°, 90°,

180°, 270° angles). No difference is expected within these categories across angle of tilt. Similarly, we expect a significant difference in response between round and square containers, with round eliciting more correct responses than square (smaller amplitude of the angle off the horizontal in children's drawings of the water-level). A significant improvement in the accuracy of the orientation with age, is also expected.

Assuming that the results from the first level analysis will allow us to discriminate clearly between the performance in 4 different task conditions, orthogonal square, diagonal square, orthogonal round and diagonal round, a number of hypotheses can be formulated as to the order of acquisition of the 4 conditions.

Hypothesis 1a:

There is a collective *décalage* between the reproduction of the water-line when the square container is oriented orthogonally and when it is oriented diagonally. orthogonal orientations are systematically solved earlier in development than diagonal orientations.

Correct performance at the diagonal orientation should be more strongly associated with correct performance at the orthogonal than with incorrect performance at the orthogonal orientations. If we illustrate the hypothesis with a contingency table with the number of correct and failed reproductions of the water-level for orthogonal orientations in the rows and the number of correct and failed reproductions of the water-level for diagonal orientations in the columns, the cell corresponding to success at the diagonal and failure at the orthogonal, is predicted to be empty under hypothesis 1a.

		Square Diagonal	
		Failure	Success
Square Orthogonal	Failure		
	Success		

Figure 5.1 Model of collective *décalage* between responses to the diagonal and orthogonal orientations of the square bottle according to Hypothesis 1a.

Hypothesis 1b:

Reproduction of the water-line when the round container is oriented orthogonally is concurrent with the reproduction of the water-line when the container is oriented diagonally.

Correct performance on the orthogonal round should be strongly associated with correct performance on the diagonal round; incorrect performance on the orthogonal round should be strongly associated with incorrect performance on the diagonal round. If we illustrate the hypothesis with a contingency table with the number of correct and failed reproductions of the water-level for orthogonal orientations in the rows and the number of correct and failed reproductions of the water-level for diagonal orientations in the columns, the cells corresponding to success at the diagonal and failure at the orthogonal, and success in the orthogonal and failure at the diagonal, are predicted to be empty under hypothesis 1b. Subjects will either fail both conditions or succeed in both.

		Round Orthogonal	
		Failure	Success
Round Diagonal	Failure		
	Success		

Figure 5.2 Model of concurrence between responses to the water-level task in the conditions round orthogonal and round diagonal according to Hypothesis 1b.

Hypothesis 2a:

The reproduction of the water-level in the orthogonal square condition is concurrent with the reproduction in the orthogonal round condition.

Correct performance on the orthogonal square should be strongly associated with correct performance on the orthogonal round; incorrect performance on the orthogonal square should be strongly associated with incorrect performance on the orthogonal round. Subjects will either fail both conditions or succeed in both.

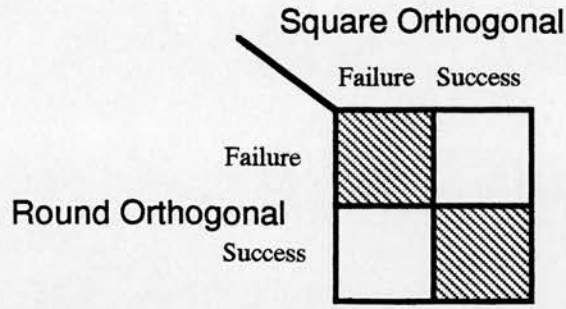


Figure 5.3 Model of concurrence between responses at the orthogonal orientations of the square bottle and of the round bottle according to hypothesis 2a (white cells are predicted to be empty).

Hypothesis 2b:

There is a collective *décalage* between the reproduction of the water-line when the round bottle is oriented diagonally and the correct reproduction of the water-level when the square bottle is oriented diagonally. Round containers are systematically solved earlier in development than square containers.

Correct performance on the diagonal square should be more strongly associated with correct performance on the diagonal round than with incorrect performance on the diagonal round.

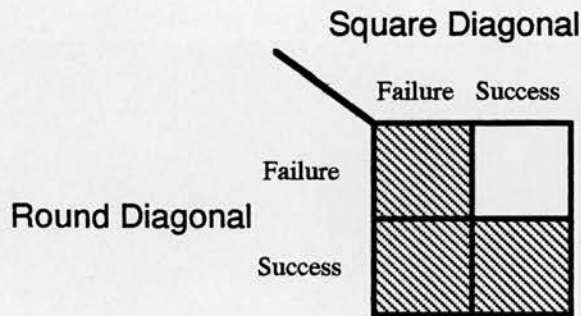


Figure 5.4 Model of collective *décalage* between responses to the diagonal Orientations of the square bottle and diagonal orientations to the round bottle according to Hypothesis 2b (white cell is the cell predicted to be empty).

Hypothesis 3a:

There is a collective *décalage* between the reproduction of the water-line when the round container is oriented orthogonally and the reproduction of the water-level when the square bottle is oriented diagonally. Orthogonal orientations are systematically solved earlier in development than diagonal orientations.

Correct performance on the diagonal square should be more strongly associated with correct performance on the orthogonal round than with incorrect performance on the orthogonal round.

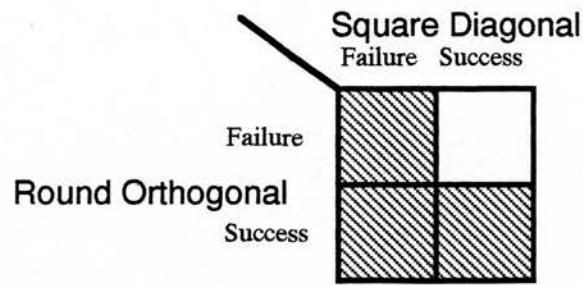


Figure 5.5 Model of collective décalage between responses to the diagonal orientations of the square bottle and orthogonal orientations to the round bottle according to Hypothesis 3a (white cell is the cell predicted to be empty).

Hypothesis 3b:

There is a relation of concurrence between the reproduction of the water-line when the square container is oriented orthogonally and the reproduction of the water-line when the round bottle is oriented diagonally.

Correct performance on the orthogonal square should be strongly associated with correct performance on the diagonal round; incorrect performance on the orthogonal square should be strongly associated with incorrect performance on the diagonal round.

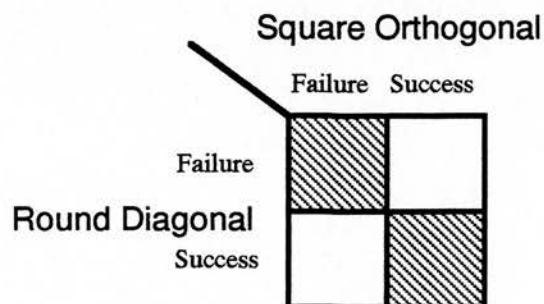


Figure 5.6 Model of concurrence between responses to the water-level task in the conditions round diagonal and square orthogonal according to Hypothesis 3b (white cells are the cells predicted to be empty).

5.4 Method

5.4.1 Design

There are two containers, a square bottle and a round bottle. There are 8 possible angles of inclination for each container. The angles are the following: 4 orthogonal tilts 0° , 90° , 180° , 270° , and 4 diagonal tilts 45° , 135° , 225° , 315° . These are the angles used by all the studies in the literature. The angles are approximated as the bottles are supported by hand by the experimenter.

Each subject will be asked to do 8 reproductions of the water-line: 4 for the round bottle, 2 orthogonal and 2 diagonal, and 4 for the square, 2 orthogonal and 2 diagonal. Therefore each subject sees each bottle in only half of the 8 possible orientations. This is done in order to keep the number of drawings per child to 8. A previous pilot experiment indicated that with children under 8 years of age, 16 drawings make the task very heavy and enhances a loss of attention. Each age group is divided in two groups presented with one of the two sets of bottle tilts.

The following patterns of presentation are used: Group 1, round(R) 0° , square (S) 90° , R 270° , S 180° , R 135° , S 45° , R 315° , S 225° . This was reversed (the round at the angles of the square and vice versa) for group 2.

















Group \ Angle	0	90	180	270	45	135	225	315
Group1								
Group 2								

Figure 5.7 Set of bottle/angle conditions presented to the two groups of subjects.

Symmetrical angles are considered equivalent. In the scoring there will be one category only covering the two orientations to the right and to the left of 0° . For example 90° and 270° will be one category, 45° and 315° another. For each bottle a subject will see only one of the two orientations in a category. Two children, one in group 1 and one in group 2 for a given bottle will see two different angles of a category, their result, however will be subsumed in the one category. If we consider

the two bottles every child will draw the water line for each of the 8 orientations. As for the order of presentation, cards were presented in a random order.

The following *independent variables* are manipulated:

- angle of orientation of containers :
 - orthogonal: 0, 90, 180, 270 degrees
 - diagonal: 45, 135, 225, 315 degrees
- shape of container : round, square
- schooling/age group:
 - primary 1 (between 5 and 6 years)
 - primary 2 (between 6 and 7 years)
 - primary 3 (between 7 and 8 years)
 - primary 4 (between 8 and 9 years)
 - primary 5 (between 9 and 10 years)
 - primary 6 (between 10 and 11 years)
 - primary 7 (between 11 and 12 years)
- set of bottle/orientations passed:
 - Group 1
 - Group 2

The *dependent variable* is the amplitude of the angle of the line drawn by the subjects to represent the water line. The drawings are later classified in correct or failed responses.

5.4.2 Statistical analysis

Two statistical treatments have been used in this study: an Analysis of Variance (ANOVA) followed by a post hoc Scheffé test, and a Prediction Analysis of Acquisition Orderings.

The first level analysis has consisted in comparing the performance of the groups for the four task conditions (round orthogonal, round diagonal, square orthogonal, square diagonal), in comparing the age groups and in comparing the performance of the two groups presented with different sets of bottle/orientation pairs.

Since children's performance was measured as the amplitude of the angles of the water line drawn inside the outline, parametric statistical tests could be used. I conducted an ANOVA to establish the significance of the effect of variables and variable interactions and subsequently a Scheffé test to identify the locus of the performance differences more precisely.

For the second level of analysis I adopted Hildebrand, Laing & Rosenthal (see Section 3.5) Prediction Analysis of Cross-Classifications method that offers a means to evaluate which hypothesis of order of acquisition of two tasks better fits the data. The 4 possible models are: concurrence, *décalage* in favour of Task1, *décalage* in favour of Task2, no systematic order between tasks or individual *décalage*.

The method consists of associating to each hypothesis a triangular model which specifies the cells that under that model are expected to be empty, i.e. error cells. For instance, in the case of the hypothesis of concurrence, a non-significant proportion of children is expected to occupy the two upper and lower corner cells, i.e. the children who succeed one task and fail the second task. In the case of collective with Task1 preceding Task2 all entries are in the diagonal cells and in the cell corresponding to success in Task1 and failure in Task2.

For each hypothesis of order between two tasks, the predictions have been tested by computing a Del index which gives the improvement over chance yield by the prediction that one or two cells are empty. This test has been presented in detail in chapter 3. The contingency tables are based on allocating children in two categories: success or failure.

5.4.3 Apparatus and Procedure

The experimental equipment consists in 2 bottles, a square plastic "laboratory" bottle sealed by a round cap with a one litre capacity and a round bottle sealed by a small cork with a capacity of half a litre. The round bottle is in fact, a glass sphere with a very small beaker. (Photographs next page). Both are half filled with red liquid.

Children are asked to draw the level of the water on a card where is depicted the side outline of the bottle. The outlined bottles are sitting on a line representing the table on which the real bottle is supported. The bottle outline is oriented to an angle. On each card there is one bottle tilted to a certain angle, this corresponds to the angle to which the real bottle will be positioned.





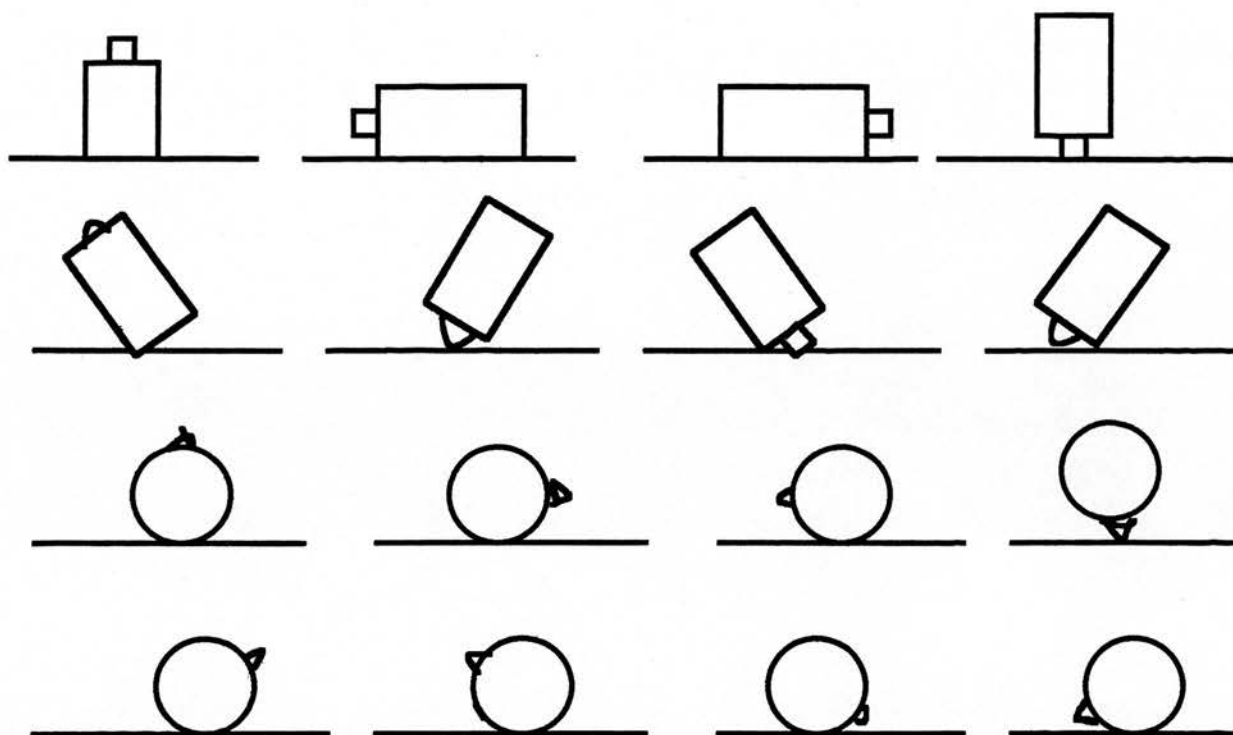


Figure 5.8 Set of 16 outlines of bottles at the various angles of tilt presented to subjects on separate cards

Children are presented with the bottles which are turned in all directions to show the movement of the liquid. The experimenter draws the child's attention on the water surface by indicating it with the finger and says : "You see the surface of the water, it is like a line, look at how the line is when I turn the bottle around." The children are given a card and the experimenter points out all the elements in the outline (the cork, the table). " Look at the picture, this is the bottle, this is the cork, and this is the table on which the bottle is sitting. " Then children are asked to draw the water line on the outline. "You see, the water is missing in the picture. Can you draw the water-line on the card?" and again "Can you show me how the water-line is when the bottle is in this position? Can you make a drawing of the water?". The child is given a red pencil to draw with. The bottle remains visible and tilted at the corresponding angle while the child is drawing⁹. The question is repeated for the 8 tilts of the bottles. Occasionally

⁹ It was decided to keep the bottle visible while the child is drawing, in order to have the most stringent measure of the child's capacity to relate the water-line to the horizontal. We are more interested in their capacity to relate the various elements of the display in a coherent representation, than in their a priori knowledge of the water-line principle. As we discussed in our literature review, the copy situation seems to be more appropriate as an orientation task, i.e. where subjects have to orient one feature of the display to the others. The anticipation task although eliciting more dramatic errors, does not distinguish clearly between the knowledge of the principle and the difficulty of finding the correct reference system in the specific situation.

the experimenter asks the child to comment his drawing or to explain verbally or with a gesture the behaviour of the water. The comments were used more as a double check of the children's answers than as data itself.

5.4.4 Interviews

The interviews occur in one of the rooms of the school, separate from the classroom, but still familiar to the children. Children are seen individually for a session of 15/20 minutes. The experimenter sits in front of the child and holds the bottles and asks the questions. A colleague sits on the side and takes notes of the subject's remarks, procedures and responses.

5.4.5 Subjects

Subjects are 140 children from a primary school in central Edinburgh. Age ranges from 5 years to 12 years, classes primary 1 to primary 7. There are 20 subjects from each class. Mean ages were the following: Primary 1 = 5.6 (SD = .29); Primary 2 = 6.7 (SD = .23); Primary 3 = 7.7 (SD = .23); Primary 4 = 8.6 (SD = .26); Primary 5 = 9.7 (SD = .25); Primary 6 = 10.3 (SD = .24); Primary 7 = 11.9 (SD = .27).

5.4.5 Measure

Responses were measured as angles off the horizontal. Measures of the subject's drawings were taken with a protractor. The orientations of the lines produced could span from 0 to 180 degrees. When lines were not drawn and children produced scribbles or coloured the totality of the figure, the drawings were scored as errors of 180 degrees. Wavy lines were approximated to the best fit, and this average line was measured with the protractor.

For the within subject analysis, any line tilted 10 degrees or less was considered correct; the 10 degrees allowing for drawing errors. Over 10 degrees off 0°, the water line was considered incorrectly oriented. In a pilot experiment, it was found that as soon as children start using lines to represent the water-line they can successfully draw the orientation of the water for the upright position of the bottle, the precision of their

drawings is of the order of 2 or 3 degrees maximum, off the horizontal. Therefore ten degrees seems a very safe margin of error due to bad motor control or other drawing skill factors.

In the contingency table, subjects were classified as failed, when they made one or more incorrect reproductions of the orientation of the water-level. This stringent criterion was applied in consideration of the fact that the bottle was always visible during the test and can be considered a facilitating situation (see chapter 4) compared to the anticipation procedure generally employed in the literature. No significant change in the results is obtained when a less stringent criterion is applied (e.g. failure attributed to subjects that produce 2 incorrect drawings per condition).

5.5 Results

5.5.1 First level analysis: mixed subjects

A 7 (class) x 2 (group) x 2 (bottle) x 4 (angle), Analysis of Variance, (ANOVA) was computed. There were 2 grouping factors, the class (P1 to P7) and the group (G1 and G2), and 2 experimental variables, the bottle shape and the 4 orientations of each bottle two orthogonal and two diagonal (where corresponding angles like 90 and 270 were considered equivalent).

Three main effects proved to be significant: Angle $F = 27.80$; $p < 0.01$, Bottle $F = 6.48$; $0.02 > p < 0.01$, Class $F = 7.40$; $p < 0.01$. There was no effect of the Group, the two sets of drawings proving to be equivalent, we will hence ignore this variable from here on.

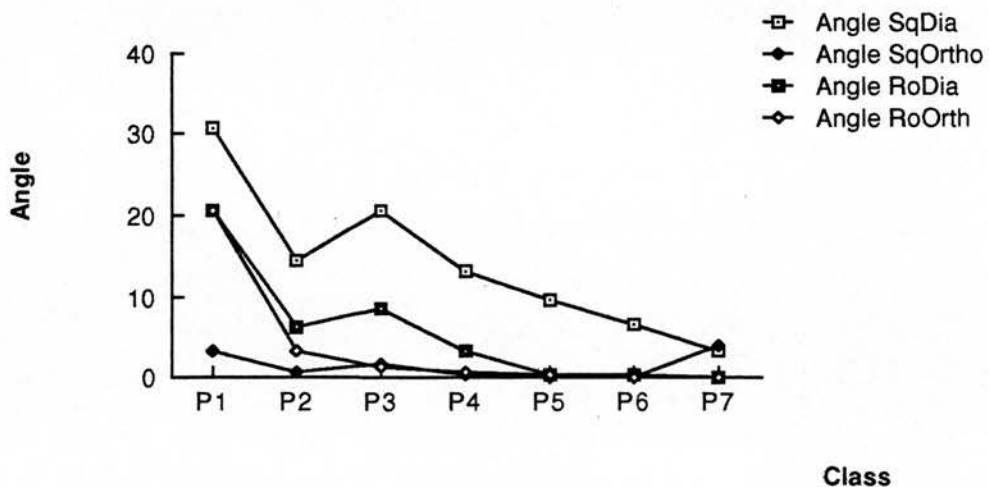
A post hoc Scheffé test was conducted to examine the source of the significant effect of the Angle of tilt. The Scheffé indicated that the source of the effect was in the difference between the diagonal angles (the 45° 315° 135° 225° angles) and the 4 orthogonals (0° 180° 90° 270° angles). No difference was to be found within the two categories. The 4 orthogonals did not differ significantly amongst themselves, nor did the 4 diagonal orientations.

Difference in the shape of the bottle proved to be significant, with the round eliciting less error than the square. An unexpected effect was the significant difference between diagonal and orthogonal angles for the round container.

While differences between classes proved significant a post hoc Scheffé test indicated that the source of difference was essentially at the level of Primary 1, which differed

from all other classes. The other classes were not significantly different from one other.

Two two-way interactions were also found to be significant. These concerned 1) angle and bottle $F = 15.05$; $p < 0.01$, and 2) angle and class $F = 2.11$; $p < 0.01$. A post hoc Scheffé test indicated that the significant differences between angles were concentrated around the diagonal positions for the square bottle. The angle x class effect seemed determined by results of primary 1 at the diagonal positions.



Graph 5.1 Amplitude in degrees of the angle off the horizontal of the line drawn by the subjects to represent the water-level. Results are presented for each class and are clustered in the 4 significant conditions of the task; Square diagonal, Square orthogonal, Round diagonal, Round orthogonal..

From this first analysis of the results we can conclude that there are in effect two distinct categories of angle of tilt of the bottles: the orthogonal and the diagonal which differ significantly between each other but not within and that the difference between round and square bottles is significant, although the difference is concentrated on the diagonal positions.

Finally an interesting result has appeared in the analysis of the age groups; with the exception of primary 1, where many subjects still scribble and therefore perform significantly worse than the other classes, the performance in the other classes is uniform. No plateau effect is obtained among the older subjects who we might have expected would draw the water-line consistently correctly. This suggests that still at the age of 12 years some children are in an intermediate stage resembling that of 7 or 8

years old children. It is therefore even more important to attempt to determine stages of development based on a non chronological criterion.

After having satisfactorily responded to our initial questions regarding the task variables, we can move to the second level of analysis where our concern will be to compare the results of each subject at the different situations.

5.5.2 Prediction Analysis of Acquisition Orderings

The testing of the specific developmental hypotheses of concurrency and *décalage* in the solution of the 4 conditions of the water-level task, requires a strict statistical analysis of children's performance across pairs of tasks. Hildebrand, Laing and Rosenthal's Prediction Analysis of Cross Classifications offers a statistical procedure to test directly the hypotheses of order of acquisition. The procedure applies to 2x2 contingency tables, where rows represent the frequency of success/failure responses to the first task and columns the frequency of success/failure responses to the second task. After each table I report the value of the Del index corresponding to the Triangular Hypothesis tested. I calculate the z score associated to the Del in that contingency table and indicate the significance level. I then compute the value of the difference between the main hypothesis and the other alternative hypotheses (z) and the relative significance level.

Hypothesis 1a: collective *décalage* between orthogonal and diagonal orientations for the square bottle.

We made the hypothesis that there exists a collective *décalage* between orthogonal and diagonal orientations of the square bottle, in favour of the orthogonal. No subject (measure error allowing) is expected to solve diagonal orientations while failing the orthogonal ones. In the language of prediction analysis we predict that Model 3, *décalage* in favour of Task1 will be significant and significantly better than the other 2 models (Model 1 concurrence between Task 1 and Task 2, Model 2, *décalage* in favour of Task 2).

The table below indicates the number of subjects having failed or succeeded the orthogonal and diagonal positions for the square container. I recall that a subject is scored as failed if he has done at least one error in reproducing the water line when the bottle was in that set of orientations.

There are 4 possible patterns of results: children failing both the orthogonal positions and the diagonal, children who succeed both orientations, children failing the

orthogonal and succeeding the diagonal and finally subjects failing the diagonal and succeeding the orthogonal. The critical case for us is failure at the orthogonal and success at the diagonal which would disprove our hypothesis of the orthogonal being a prerequisite for the success at the diagonal.

		Square Diagonal.	
		Failure	Success
Square Orthogonal.	Failure	1	0
	Success	95	44

Table 5.1: Contingency table of success and failure to orthogonal and diagonal orientations for square bottle

Model 3, décalage in favour of Task 1, i.e.orthogonal square, has proved significant. The Del is equal to 1 and the model is a significantly better predictor than than any of the other two models, concurrence or the opposite collective décalage.

Mod 3: Del = 1 $z = (E) p < 0.00001$

$z_{M1-M3} = -144.8; p < 0.00001$

$z_{M2-M3} = -288; p < 0.00001$

Our hypothesis that correct reproductions of the orientation of the water-line when the bottle is tilted at an orthogonal angle appears prior to the correct reproductions of the orientation of the water-line when the bottle is tilted at an diagonal angle, has been confirmed. It is also interesting to remark that only 1 subjects out of 140 was not capable of reproducing the water-line correctly in either conditions.

Hypothesis 1b: concurrence between orthogonal and diagonal orientations for the round bottle.

We made the hypothesis that there exists a relation of concurrence between the solution to the orthogonal and diagonal orientations of the round bottle and we expect that subjects will either succeed or fail both task conditions. In the language of prediction analysis, we predict that Model 2, concurrence between Task1 and Task 2, will be significant and a significantly better predictor than the other 2 models.

The table below indicates the number of subjects having failed or succeeded the orthogonal and diagonal positions for the round container. A subject is categorised in

the failure group if he has done at least one error in reproducing the water line when the bottle was in that set of orientations.

There are 4 possible patterns of results: children failing both the orthogonal positions and the diagonal, children who succeed both orientations, children failing the orthogonal and succeeding the diagonal and finally subjects failing the diagonal and succeeding the orthogonal. The critical case for us is that one of the cells indicating failure at one condition and success at the other should receive a high frequency, which would disprove our hypothesis of concurrence.

		Round Diagonal	
		Failure	Success
Round Orthogonal.	Failure	11	1
	Success	34	94

Table 5.2: Contingency table of Success and Failure to orthogonal and diagonal Orientations for round Container

The hypothesis of concurrence was not confirmed. Model 2 was not found to yield significant improvement over chance. It is clear from this table that a significant number of subjects, succeed in the orthogonal condition while failing the diagonal condition. On the contrary just 1 subject succeeded at the diagonal while failing the orthogonal. Alternative hypotheses were examined and Model 3 was tested to verify whether a collective *décalage* in favour of Task 1 (orthogonal condition) is holding between the two conditions.

In the case of the round container Model 3, *décalage* in favour of Task 1, orthogonal round, proved significant. The Del index is equal to .87 and is significantly better than Model 1 or 2.

Mod 3 : Del = .87 $z = (7.1)$ $p < 0.00003$

$z_{M1-M3} = -5.4$ $p < 0.00001$

$z_{M2-M3} = -6.3$ $p < 0.00001$

We can conclude therefore by saying that for both round and square containers the performance at the orthogonal is a good predictor of performance at the diagonal, i.e. children who fail the diagonal also fail the orthogonal, while children who succeed the orthogonal may either succeed or fail the diagonal.

Hypothesis 2a: concurrence between orthogonal orientations of square and orthogonal orientations of round containers.

The hypothesis that the orthogonal angles are solved concurrently in both containers is evaluated by children's performance in the task conditions orthogonal square and orthogonal round. The model which is expected to provide the best fit is be Model 2 of concurrence between tasks.

		Square Orthogonal.	
		Failure	Success
Round Orthogonal	Failure	0	12
	Success	1	127

Table 5.3: Contingency table of success and failure to orthogonal square and orthogonal round conditions.

The table presents a very unbalanced distribution. The vast majority of subjects (90% of subjects) solved both tasks correctly, no subject failed both task, only very few subjects failed one task while succeeding the other. Such a distribution does not allow to test any prediction and the triangular test of these three relevant developmental hypotheses is meaningless. None of the models was found to be significant:

Mod 1: Del = -.014 ; Mod 2 : Del = -.8 ; Mod 3 : Del = -.93

While unable to confirm the hypothesis that the two conditions are acquired concurrently (for this we should test younger subjects that would fail both tasks) these results suggest that there is a stable stage in which subjects can solve at the same time both orthogonal conditions while still failing the diagonal positions of the two bottles.

Hypothesis 2b: collective décalage between diagonal orientations of the round container and diagonal orientations of the square container.

In the comparison of the diagonal orientations for the two bottles, we made the hypothesis that the round bottle would be found easier and that the model of collective décalage in favour of the round diagonal would be found significant.

		Square Diagonal	
		Failure	Success
Round Diagonal	Failure	39	6
	Success	57	38

Table 5.4: Contingency table of Success and failure to diagonal square and diagonal round conditions.

The hypothesis of *décalage* is confirmed. Model 3 is significant, and is a better predictor than other other two models.

Mod3 : Del = .575 $z=3.8$ $p < 0.00003$

$z_{M1-M3} = -3.4$ $p = 0.00034$

$z_{M2-M3} = -3.65$ $p < 0.00016$

In the diagonal condition, the difference pattern of development of horizontal in the round and square containers, is especially clear. A considerable proportion of subjects are able to correctly reproduce the orientation of the water-level when the round container is tilted at some diagonal angle while at the same time still failing the reproduction of the water-level when the square container is tilted at the same angles.

Hypothesis 3a: collective *décalage* between square diagonal and round orthogonal conditions.

The hypothesis was made that there is a collective *décalage* between correct responses in the condition round container oriented at orthogonal angles and square container oriented at diagonal angles. In the language of Prediction analysis this corresponds to the hypothesis that Model 3 collective *décalage* in favour of Task 1 (round orthogonal) will provide the best fit for the data.

		Square Diagonal	
		Failure	Success
Round Orthogonal	Failure	12	0
	Success	84	44

Table 5.5: Contingency table of success and failure to orthogonal round and diagonal square Conditions

Model 3, décalage in favour of Task 1, i.e.orthogonal round, has proved significant. The Del is equal to 1 and the model is a significantly better predictor than than any of the other two models, concurrence or the opposite collective décalage.

Mod 3: Del = 1 $z = e$ $p < 0.00001$

$z_{M1-M3} = -62.7$ $p < 0.00001$

$z_{M2-M3} = -97.8$ $p < 0.00001$

A developmental order holds between succeeding at the orthogonal orientations of the round container and succeeding at the diagonal orientations of the square container.

This does not come as a surprise as it is by now becoming clear that the diagonal positions of the square bottle are by far the most difficult condition and the last to be acquired.

Hypothesis 3b: concurrence between square orthogonal and round diagonal conditions.

An initial hypothesis was made predicting that there is a relation of concurrence between the correct reproduction of the water-line in the orthogonal orientations of the square bottle and the diagonal orientations of the round container. In the language of prediction analysis this hypothesis corresponds to predicting Model 2 concurrence between task 1 and task 2, is the best predictor of children's performance in the two tasks.

		Square Orthogonal	
		Failure	Success
Round Diagonal	Failure	1	44
	Success	0	95

Table 5.6: Contingency table of success and Failure to orthogonal square and diagonal round Conditions.

The hypothesis of concurrence was not confirmed. Model 2 was not found significant. It is immediately evident from this table that a significant number of

subjects, succeed in the orthogonal square condition while failing the diagonal round condition. On the contrary, no subject succeeded at the diagonal while failing the orthogonal. The hypothesis was revised and Model 1 was tested to verify whether a collective *décalage* in favour of Task 2 (orthogonal square condition) is holding between the two conditions.

In the case of the round container Model 1, *décalage* in favour of orthogonal square proved significant. The Del index is equal to 1 and is significantly better than Model 2 or 3.

Mod 1 : Del = 1 $z = e$, $p < 0.00001$

$z_{M1-M2} = 144.8$ $p < 0.00001$

$z_{M1-M3} = 288$ $p < 0.00001$

A very strong effect was thus found showing the anticipation of success at the square orthogonal condition over the diagonal round.

In conclusion, we can summarise the results of this hierarchical analysis in a schema which describes the developmental sequence encountered:

Level 1) failure in both orthogonal and diagonal orientations for all shapes

Level 2) Success at all the orthogonal positions for round and square containers

Level 3) success at both diagonal and orthogonal orientations with the round container

Level 4) success at both diagonal and orthogonal orientations with the square container.

The first acquisition to emerge is the capacity to reproduce the water-level in the orthogonal positions of both containers, and this with a clear anticipation on the diagonal orientations. Contrary to our initial hypothesis the round diagonals are found more difficult than the round orthogonals. This same pattern is observed in the fact that while the solution of the orthogonal conditions emerge concurrently with the square and round containers, an order of acquisition in favour of square orthogonals hold between the latter and the round diagonals. However the diagonal orientations of the round container are solved correctly before the diagonal positions of the square bottle. An interesting observation is that within our sample there was only 1 subject that failed all conditions, that could be classified as a level 1 subject. All the subjects, apart from him, were successful in depicting the correct orientation of the water-level when the square bottle was in one of the 4 orthogonal positions. On the contrary 11 subjects failed both conditions with the round container.

A few words should be spent on the type of errors children are performing. The first noticeable fact is the small number of inversion errors, that is those drawings reported by Piaget in which the water-line is represented as parallel to the bottom of the

container as though it stays stuck to the bottom. Ford and Thomas & Jamison had already remarked that in the copy task, when children can see the bottle while drawing, this type of error is less frequent. This experiment confirms their finding. Most of the errors encountered are of the order of 20 to 40 degrees off the horizontal, and could be classified according to Piaget's categories as stage IIIa errors. They consist in fact of lines drawn slightly diagonally a much less dramatic error than the inversion errors, though they manifest a clear uncertainty as to the direction and orientation that the water-line should receive.

5.6 Discussion of the results

The data collected in this experiment has confirmed some of our hypotheses, disproven others but globally provided interesting new evidence on children's competence in orienting the water-line. I will initially discuss the various hierarchies established separately for each condition, and then consider the global sequence that emerges from this experiment.

The first comment should bring on the clear distinction that has emerged between the performance in the different conditions of the task. Significant differences were found in the reproduction of the water-level between round and square containers and between orthogonal and diagonal orientations. It must be stressed again, that within the two categories of orientations no significant differences were identified. In particular, the type of order classically described between orientation 0° and 90° , has not been found. All the children are capable of drawing the water-line horizontally for all 4 orthogonal positions at the same time. These four orthogonal positions can be classified as a specific category which opposes itself to the category of the four diagonal orientations. This distinction is an important element in determining the developmental steps leading to a generalised correct representation of the orientation of the water-line.

5.6.1 The orthogonal effect with the square bottle

It has appeared that there is a direct transition from scribbling, to drawing the water-level correctly in the 4 orthogonal positions. It is as though as soon children start representing the water with a line they are capable of orienting it correctly in those positions. In fact hardly any children (only 1 out of 140) represented the water in the

square bottle with a scribble, and all of them apart this single child are immediately correct in their orientations of the water-line for the orthogonal positions. This was probably determined by the the age group chosen in which all subjects were able to perform line drawings. On the basis of the pilot study I had in fact excluded nursery children who were found to be unable of representing the water with a line. However for the round sphere, there were a few more subjects (12 subjects all in Primary 1 and Primary 2) who made errors in the orthogonal positions as well as the diagonal positions. What is extremely interesting is that these subjects are incorrect because they represent the water with a scribble and not because they orient the line depicting the water at the wrong angle. In the orthogonal positions of the square bottle the same 12 subjects draw the water-level as a line oriented horizontally, while they draw the water as a scribble when presented with the round bottle (and are thus scored as failing the task). They seem to regress to a more primitive style of drawing when faced with the round bottle. A preliminary conclusion can be drawn, which will need some confirmation by comparing a number of nursery children to primary school children, is that in the transition from scribbling to line drawings, children immediately start orienting the line horizontally in the orthogonal conditions. In conclusion the important fact that has emerged is that no subject drawing the water-level with a line, orients the line incorrectly for any of the orthogonal positions of the containers. If error there is in this condition, it consists in drawing the water as a little shapeless scribble or consists in drawings that "fill up" the whole bottle.

This finding along with the clear developmental order established between the orthogonal and diagonal orientations of the square bottle, goes along with the initial hypothesis of a significant difference in the two conditions of the classical task. The orthogonal positions all have in common the perpendicularity of the water-line with some element of the square bottle, which may explain why they are all solved simultaneously. This suggests that at this stage subjects are representing principally the spatial relations holding between the water-line and the bottle and not between the water-line and an external frame of reference. Observing more closely children's errors in the diagonal positions of the bottle, children seem to be attempting to establish the correct relation between the water-line and the sides of the bottle. I have often observed children trying to measure the distance of the water-line from the beaker or the distance of the point of conjunction of the line with the side from the bottom of the bottle. They appear to be trying to draw a line between two specific points on the sides of the bottle, what may be interpreted as an attempt to relate the water-line exclusively with the bottle and not with external referents.

5.6.2 The orthogonal effect with the round container

The most striking result of this experiment is undoubtedly the difference in children's performance between the diagonal and orthogonal positions of the round container and the clear collective *décalage* in favour of the orthogonal orientations that holds between these two conditions. Apparently, if we disregard the position of the little spout, there are no geometrical differences in the glass sphere when it is oriented at 45 or 90 degrees. In fact if the spout was not there, there would be no index at all of the rotation of the sphere; no index of the transformation in the spatial relation of the sphere with the environment. Furthermore there is no evident transformation of the relation between the water-level and the bottle. The different orientations of the spout do not affect the overall configuration of the display and the symmetry of the round shape keeps the relation between the water and the parts of the sphere, equal.

The fact that subjects are so sensitive to the change of orientation of the sphere that they orient the water line horizontally more easily when the spout indicates that the sphere has been rotated at 90, 180 or 270 degrees, is an extremely interesting result. The only possible interpretation of these results is that children are structuring the figure in a much more sophisticated way than expected, and are attributing some internal axes to the sphere.

In an interesting study on the representation of the orientation and form of objects, Rock (1973) claimed that a fundamental component in the recognition and representation of objects is the assignment of directions such as top, bottom, left and right. Moreover, Arnheim (1974, p.13) suggested that objects have hidden structures consisting in the major horizontal and vertical axes and major diagonals. I suggest that subjects faced with the round container attribute a canonical top to the sphere on the basis of the position of the spout, and derive the major axes of the object by drawing an imaginary line running through the spout and joining the bottom of the sphere.

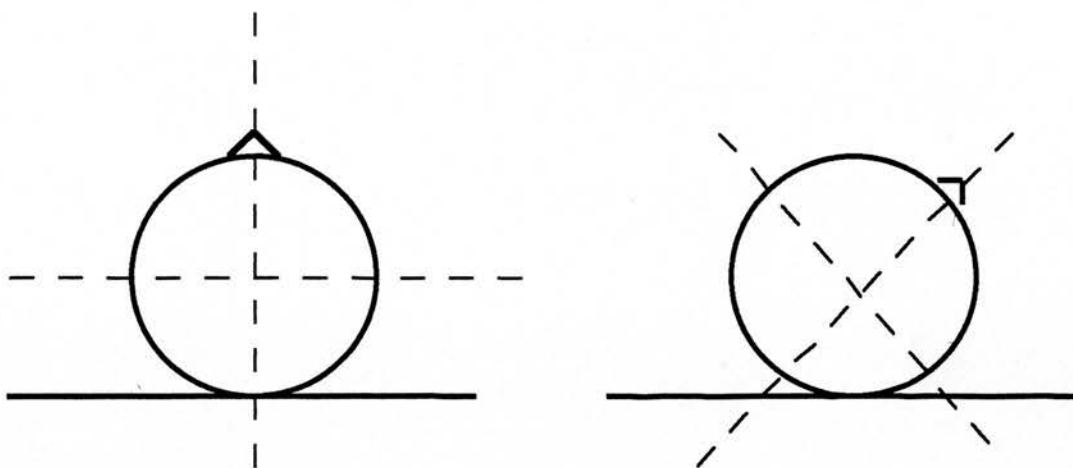


Figure 5.9 The major axes of the spherical container

We can suppose that first of all, children are attributing a canonical top and bottom to the container. The spout probably comes to constitute the top of the sphere, while the bottom may be the part opposite the spout, and determines the main axes of the object. The consequence of this attribution of axes and directions to the sphere, is that these axes become the internal referents for the orientation of the water line. If the internal structure of the sphere is organised around two main perpendicular axis, these axes can become the referent for parallelism and perpendicularity. It is therefore on the basis of these axes that the rotation of the object is identified and more important the orientation of the water-line is represented.

Let us consider the case of the orthogonal orientations of the sphere. The water-level is parallel to the major axes of the sphere: the vertical axis, when the sphere is tilted at a 90° angle; the horizontal axis, when the sphere is upright or tilted at 180° . Therefore when the water-level coincides with one of the axes, the children represent the level horizontally.

In the diagonal tilts of the sphere, the water-level is neither parallel nor perpendicular to the orthogonal axes of the sphere. If one considers the errors children do when the sphere is in a diagonal position, these are very similar to those for the square container. The same 20 or 30 degrees errors are observable, the same search for the correct distance from the spout. Children seem to have an identical difficulty in determining the angle of incidence (an oblique angle) of the water-line with the orthogonal axes of the sphere as with the straight sides of the square bottle.

The few inversion errors that have been found, can also be explained by children's reliance on these internal axes of the sphere. When the water-line is drawn opposite to the beaker as a perfect perpendicular to the main axis going through the spout, we can only marvel at the subjects' skill in rotating their representations and the ability to

preserve the spatial relation that they consider pertinent, that of the water-line with the vertical and horizontal axes of the canonical bottle.

Piaget's interpretation of children's performance with round containers according to which they allow more easily to "brake free" from the internal frame of the bottle and consider the reference offered by the table top, is therefore seriously challenged by the *décalage* between the two set of orientations. On the contrary, these results give additional support to the hypothesis that at a first stage children are representing the water-level horizontally only when the level is also perpendicular or parallel to some of the elements internal to the bottle both when they are "given" as in the square bottle and when they are "constructed" in the round bottle.

5.6.3 The order between orthogonal square and diagonal round.

According to the previous interpretation of children's performance with the round container, it comes as little surprise that the orthogonal positions of the square bottle are correctly solved before the diagonal positions of the round sphere. If initially subjects rely on the perpendicularity or parallelism of the water-level with the internal elements of the container, be this the side of the bottle or some major axis, the order relation holding between orthogonal square and diagonal round is no different from the one holding between the orthogonal and diagonal orientations for the square bottle alone. Thus the category distinction orthogonal - diagonal is just as effective even though the shapes are different. This result reflects again the clear distinction typical of a certain stage of development, between situations in which the horizontal is determined by relying on the internal relation of the water with the container and situations in which this internal relation does not allow to solve the problem.

5.6.4 The order between diagonal round and diagonal square

The question of the anticipated solution of the round diagonal orientations before the square diagonal is more complex than we had envisaged when initially the hypothesis had been made that round containers in any orientation are concurrent with the square orthogonals. It is undeniable that the round container is found easier than the square when they are both in any of the diagonal positions. In fact while 67% of the subjects succeed in the diagonal position for the round bottle, only 31% of the subjects performed accurately with the diagonal square. It seems therefore that, although children make the distinction between the orthogonal and diagonal positions of the

sphere, eventually the difficulties with the latter set of orientations is overcome more easily than in the case of the square bottle. Alongside the facilitatory effect of the angle of tilt we must therefore, posit a shape effect.

This result combined with the *décalage* between orthogonal and diagonal orientations of the sphere, puts us in a paradoxical situation. In fact, in view of the distinction between orthogonal and diagonals that the children make, we cannot invoke an explanation simply based on the invariance of the configuration of the sphere. It seems evident from our previous discussion that children are in fact putting a lot of structure in the round figure, therefore when they solve the diagonal positions they are genuinely achieving a novel solution. However the *décalage* between the diagonal round and square does not allow us to explain their performance solely in terms of the effect of the angles of tilt of the bottle. Anticipating slightly on the general interpretation I will be proposing in the next section, the suggestion can be made that the anticipated discovery of the invariance of the water level in the sphere is to be explained by the symmetry of the figure which allows children to determine the more easily the position of equilibrium liquids achieve within containers.

5.6.5 The general hierarchy

It is interesting to remark that when we start considering children's solutions to tasks from a hierarchical perspective and we start identifying levels of concurrent solutions, the perspective on the problems change. In fact the question becomes one of explaining what conceptual relation could relate factors as shape and orientation. In particular, in the case of concurrent solutions of the round and square orthogonals, the question is raised of what is the nature of the child's theory that can produce correct reproductions of the water-level and at the same time, fail with the round diagonal? When we start considering ordered levels of success and relate the situations where these successes occur the issue shifts from one of explaining the errors to one of explaining both the successes and the errors.

The results with the sphere have brought significant new elements to interpret the correct reproductions of the water-level in the orthogonal positions of the square bottle. In fact, contrary to Piaget who claimed that these productions were the first attempts to relate the water level with an external reference for horizontality, the evidence of anticipated success at the orthogonal positions of the sphere over the diagonal positions, suggest that at this stage children are essentially concentrating on the relations internal to the bottle. If subjects are structuring the sphere in terms of axes,

tops and bottoms etc. in order to refer the water-level to its internal elements, we can suppose that the square bottle is being similarly structured and is providing an internal frame for determining the orientation of the water. Furthermore the errors of 20 or 30 degrees committed by children in the diagonal positions of the containers can be seen as a consequence of the same phenomena. If subjects are treating the problem as one of referring the water-line to the sides and axes of the container, the angular orientation of the water-line with respect to these elements becomes extremely difficult to represent. There is an extensive literature (Olson 1970, Bryant 1974) showing that children are very poor at representing oblique lines .

The most revealing fact that has emerged from this study is the complex nature of the intermediate subjects' representations. The relations established between the water-level and the internal elements of the bottle are highly structured and demonstrate an organised spatial system operating within the "space" of the object. This system is by far more sophisticated than what Piaget had characterised as a transition phase from topological to euclidian spatial representations. Not only are children establishing the correct orientation of the water line but they are succeeding in doing so by structuring the objects involved in the picture according to a complex system of euclidian relations involving axes, parallels, perpendiculars as well as rotations. That these relations are essentially centered on the elements within the bottle does not diminish the complexity of the spatial relations employed. To characterise these representations centered on the "internal" relations we can invoke the notion of an "object centered" geometry, a notion that has been proposed by Olson & Bialystock (1983) to account for these type of representations.

5.6.6 A theory of object centered geometry

In their book "Spatial Cognition" (1983) Olson & Bialystock, propose a reinterpretation of Piaget's data on the water-level problem, based on their theory of the development of spatial representation. Olson & Bialystock suggest that there are various categories of objects or entities which can serve as references for defining spatial relations between objects: egos, canonical objects and noncanonical objects. Canonical objects such as bottles, tables, cars, or houses are attributed a canonical top, bottom, front and back and intrinsic spatial axes such as horizontal and vertical. These spatial properties define in certain sense the representation of the canonical object and allow its identification. Canonical objects have parts that are assigned intrinsic spatial

properties such as tops bottoms etc. Once assigned these intrinsic spatial descriptions become part of the object and their specification does not depend any longer on the particular position or orientation of the object as a whole. In a way, these canonical objects or their parts can become referents for the position or orientation of other objects. For instance when we say that "the pen is at the bottom of the bag" we are identifying a location by referring to the "space" of the bag. Canonical objects become therefore references for spatial relations between objects.

Alongside canonical objects, Olson and Bialystock define noncanonical objects. These are objects in the environment which do not have any particular spatial orientation and no particular feature marking spatial position. Frameworks or displays are a particular type of noncanonical objects. These spaces are assigned temporary spatial properties such as tops and bottoms and horizontal and vertical axes. Once assigned a set of spatial properties, these displays are treated as canonical objects. The temporary assignment of these spatial attributes to noncanonical objects may depend either on other canonical objects or on ego related space (ego is the third type of spatial reference that individuals can employ). Overall however, the ultimate referent for verticality, and consequently for horizontality, is gravity.

According to these authors in order to solve the water-line task, the water-level orientation must be coded relative to a horizontal axis in the sense that the water level must be represented as being *on* the horizontal axis. They argue that the problem subjects are faced with, is that of identifying which horizontal axis is appropriate. The horizontal axis derived from the object space of the bottle or the primary horizontal axis of the display? They argue that there is a predominant horizontal axis intrinsic to the bottle and that this enters in conflict with the horizontal axis of the display. It is the conflict between these two horizontal axes that according to Olson and Bialystock gives rise to the typical errors and in particular the inversion errors:

It is interesting to note that Piaget and Inhelder described the use of the first horizontal, what we have called object space, as being determined by the particular configurations presented. They claim that horizontal and vertical axes are still undiscovered (p.382 Piaget & Inhelder 1956). While we agree that the child is not using "external" frames of reference, their responses nonetheless systematically honour horizontal and vertical axes. Their axes however, are defined on the basis of the objects themselves. The same explanation applies to Piaget's & Inhelder's observations that children position fenceposts and houses perpendicular to the hill rather than perpendicular to the external frame of reference; both cases involve the propositions of perpendicular to the Horizontal axis; the only question is which Horizontal axis is relevant. (1983 p. 241)

In sum, in the case of the water-line task, therefore, the conflict arises between choosing the bottle (a canonical object) as referent and choosing the display (a

noncanonical object) as referent. It is because children do not know the invariance of orientation of liquids, claim the authors, that they choose the incorrect referent.

While agreeing with the general lines of this interpretation, it must be pointed out that the phenomenon Olson & Bialystock wanted to explain was principally the inversion errors described by Piaget, where the water-level is drawn always parallel to the bottom of the bottle. For this type of errors their explanation is perfectly satisfactory. However, something more should be added to interpret the findings that children succeed in the 90 degrees condition and draw diagonal lines when the bottle is tilted at a diagonal angle. In the case of the correct drawings at the 90 degrees orientation the referent axis could not be the canonical horizontal axis of the bottle. In the same way children who draw incorrect diagonal lines when the bottle is tilted are not referring their line to the canonical horizontal axis. These behaviours raise the question of the role played by the canonical axes in establishing the orientation of the water. We may suppose that rather than behaving as an invariant internal horizontal and vertical, they are more simply spatial "anchors" to guide the orientation of the line representing the water. In fact in the case of the orthogonal positions of the bottles, the axes rather than creating a conflict between the internal and external axes, seem to help the correct orientation of the water-line.

If Olson & Bialystock are correct in claiming that objects come to be structured in a system of top and bottom and of axes, these objects somehow become specific "spaces" within which other elements are referred. This is probably particularly true for objects which are containers. The axes can therefore help to organise this space and especially can serve as an internal system of reference on which are anchored the orientations of the various elements of the object. The axes therefore work as an internal system of coordinates. The question remains as to whether this system of reference has a fixed interpretation as Olson & Bialystock would suggest; whether they are attributed a direction of verticality and horizontality once and for all or whether they simply are reassigned directions of verticality and horizontality for each new position of equilibrium.

We may consider that the two major perpendicular axes are not given a fixed interpretation of horizontal and vertical but that the directions are attributed according to the position of equilibrium of the bottle. If the bottle is tilted on its side at 90 degrees, the major axis connecting the canonical top and bottom of the bottle is interpreted as a horizontal axis, while when the bottle is upright the same axis is interpreted as a vertical axis. The horizontal axis of an object is the axis that is perpendicular to gravity when this object is in equilibrium. In the case of the orthogonal positions of the bottle the axes of the bottle coincide with the axes of the

horizontal and vertical of the environment, while there is no such correspondence when the bottles are in a diagonal position. In the latter positions therefore, the internal axes of the bottle are of no real aid in determining the orientation of the water-line. They may even be prejudicial as it is well known how difficult it is for children to orient a diagonal line with respect to another tilted line (as are the internal axes when the bottle is for instance at 45 degrees).

In the water line task, we are thus confronted with a strange situation in which first a child attributes a horizontal axis to the bottle on the basis of its perpendicularity to gravity when the bottle is in equilibrium in its canonical position, then the bottle itself becomes a referent for horizontality. The child therefore judges or attempts to judge the orientation of the liquid from the frame offered by the bottle rather than referring to the direction of gravity that was the initial referent for attributing the main axes to the bottle. What will determine that eventually the child will go and search for the reference of gravity to determine the position of the water? Possibly the fact of realising the dynamic independence of the water from the bottle, or the lack of correspondence between the direction of the axes of the bottle and the direction of the horizontal and vertical in the environment. In any case the discovery of some of the physical properties of liquids must have an important role.

5.6.7 Concluding questions

This interpretation opens a number of questions on the transition from an object centered geometry to a framework centered one. In fact a lot of data concurs in suggesting that in the case of the water-level task the choice of the correct external reference emerges as a consequence of knowing the physical properties of liquids. It is very difficult on the basis of this task to decide whether the object centered to framework centered shift we observe in our subjects corresponds to a general transformation in their spatial representations, a transformation which we could observe in a series of different situations. In other words, can we characterise object centering as a general stage in children's spatial representations; a spatial system which will be subsequently substituted by a framework centered system? Alternatively we may consider that both systems are available to children and adults alike and that the reliance on one system or the other depends on the specific constraints of the situation.

It is unlikely that we could answer a question of this amplitude in this thesis. However by introducing a new situation in which the physical setting is modified, a

first step can be taken to investigate the generality of children's reliance on internal references. The next experiment will compare children's performance in the water-level problem with a similar task which involves the orientation of the crossbar in a balance scale.

Chapter 6: Experiment 2

6.1 Introduction

The results from the previous experiment have led us to advance the hypothesis that there is a stage in the development of horizontality, in which children are "object centered". When they have to represent the orientation of a surface or a part of an object, they rely on the features internal to the object to represent and orientation the plane. In other words, the only salient spatial relations are those established by the plane with the other parts of the object. This stage is followed by a stage in which children are capable of establishing the orientation of a plane independently from its relations to its contiguous elements, but rather by relying on external references given by the direction of gravity and the orientation of other objects in the environment.

In order to verify the generality of this phenomenon and to evaluate the role of the specific physical knowledge involved in the water-level problem, an alternative situation is introduced. The physical complexity of the water-level problem, proof of which is the poor results obtained with adult subjects, suggest that we verify children's competence in representing the orientation of horizontal planes with different materials. An experimental setting must be envisaged that while preserving the spatial structure of the water-level task, does not make appeal to the complex dynamics of liquids. As previously suggested, it may be the case that the children experience great difficulty in taking into account the dynamic properties of liquids and for that reason, reduce the problem to one only involving the relations liquids establish with their containers.

The new equipment tested in this second experiment, was suggested by the Crossbar experiment of DeLisi, DeLisi & Youniss (1977) presented in section 4.4.3, with the difference that rather than having an apparatus that maintains the crossbar horizontal by some arbitrary properties of the pivot point, a balance was employed. Similarly to the crossbar apparatus the balance has a T shape and is constituted of three parts: a base which supports a bar (which I shall call the pivot) and a crossbar. The crossbar can swivel on a pivot-point that joins it to the pivot, as in any balance. Furthermore, the pivot itself can also swivel on a pivot-point that joins it to the base. The pivot is mobile and can tilt at an angle with respect to the base of the balance (e.g. it can form an angle of 45 degrees with the base).

The crossbar is constructed so as to be able to support two small objects at each of its two extremities; when the two objects are the same weight, the crossbar stays

horizontal regardless of the inclination of the pivot. In fact since the crossbar is dynamically independent from the pivot, it stays horizontal when an equal pull is exerted on its extremities. Two equivalent weights determine the equal pull and the invariant position of the crossbar.

From an early age children have experience with scales, phenomena of balancing, and weighing objects. The balance task, tested in a pilot study on young children, seemed to be a familiar situation in which children were particularly good at predicting what happens when equal or different weights are put on the scales. The horizontality of the crossbar may be seen as more meaningful than horizontality of the water level as it is informative of the fact that the two objects on the balance are equivalent in weight. While in the water-level task the horizontal *describes a state* of the water, the horizontality of the crossbar is the *index* to determine the equivalence of the weights.

The spatial requirements of the balance task seem to share the same underlying structure of the water-level task. As in the classical water-level task, the balance involves the orientation of a plane which stays invariably horizontal by physical necessity. Both the balance task and the water-level task involve three objects: for the former these are the crossbar which stays horizontal and therefore is dynamically independent from the "body" of the balance, the pivot which modifies its spatial relation with the environment, and the environment which gives an external frame of reference and with respect to which the pivot changes position. As in the water-level problem, a change in the relation between the pivot and the environment entails a modification in the relation holding between the pivot and the crossbar, while the relation between the crossbar and the environment stay invariant.

In this task as in the water-level task I wish to contrast two conditions the orthogonal orientations and the diagonal orientations of the pivot-bar. In the previous experiment it appeared that when the container was oriented orthogonally the water-level orientation was represented correctly before children could also orient correctly the water-level when the container was in the diagonal positions. This phenomenon was explained by the fact that orthogonal positions of the container could be solved by referring the water-level to the internal axes of the container. The diagonal positions on the contrary, required that children abandon the internal reference and take into account the orientation of the water-line with respect to the environment. Similarly the balance task will contrast a situation in which the crossbar is perpendicular to the pivot and parallel to the base, to a situation in which the perpendicularity is broken and the crossbar is at angle with its supporting pivot. The orthogonal condition of the balance can be solved by establishing only the relations between the different elements of the object while the diagonal condition of the balance makes it necessary to consider the

orientation of the crossbar with respect to the total display. If there is a general tendency to determine the orientation of a plane first by focusing on the relation this plane establishes with its contiguous elements and then with respect to the external framework, the orthogonal orientations of the pivot should be solved before the diagonal orientations. It is therefore predicted that a similar pattern of development will be found in the balance task and in the water-level task, with the orthogonal conditions succeeded prior to the diagonal conditions.

6.2 Objectives

The first objective of the experiment is to verify whether there is a collective *décalage* in the solution of the two conditions of the balance task, whether in other words, the orthogonal orientation of the balance is solved before the diagonal position of the balance. The hypothesis proposed in the conclusion of the previous experiment, that there is a stage in which children rely on the internal features of the object to determine the orientation of a plane, predicts that subjects will succeed in the orthogonal condition of the balance task while failing the diagonal condition (as they succeeded in the orthogonal positions of the bottles while failing the diagonal orientations).

The second objective is to replicate the findings of the previous experiment on the water-level with the two containers, round and square. The same patterns of collective *décalages* and concurrences between the various conditions found in the previous study, should be confirmed in this experiment.

Finally the third objective of this experiment is to explore the relationship between the solution of the balance and water-level tasks and in particular whether the solution of the balance task precedes or is concurrent with the solution to the water-level problem. If there is a general stage in which children relate the positions of a part of an object simply to the other parts of the object; i.e. they are "object centered", we may make the hypothesis that the orthogonal position of the balance will be acquired concurrently with the orthogonal positions of the bottles. All these solutions should precede the correct solutions of the respective diagonal orientations.

As far as the shift to the correct representation of the horizontal plane when the balance or container is oriented diagonally, there are two hypotheses that can be advanced. If an order is established between the diagonal and orthogonal conditions with the balance the hypothesis may be made that the acquisition of the diagonal for the balance may co-occur with the acquisition of the diagonal positions of the containers.

In this sense the transition from an internal to an external frame of reference would be general and synchronous across different situations.

However we have observed in the previous study that there is a *décalage* between the round and square containers for their diagonal positions, this already suggests that there are facilitatory contexts allowing children to take into consideration the external frame of reference before other contexts. The most plausible hypothesis is therefore that there is a *décalage* in the solution of the diagonal positions of the balance and the diagonal positions of the bottles. I would also propose that the *décalage* is in favour of the balance as it constitutes a more familiar situation of which children may have better grasp. The physical principles involved in maintaining the crossbar horizontal, although complex, may be better understood by the children and thus allow them to envisage more easily the dynamic independence of the crossbar and its invariant horizontality. The hypothesis is thus advanced that the acquisition of the diagonal positions of the balance precede the acquisition of the diagonal with the bottles. In conclusion, by modifying the physical setting of the task and proposing a situation that should be simplified with respect to the water-level problem, it is assumed that the correct orientation of the horizontal plane constituted by the crossbar will be represented before the horizontality of the water-line.

6.3 Hypotheses

The first type of hypotheses regard the differences between task conditions. For the three tasks, reproduction of the water-level in the square bottle, reproduction of the water-level in the round container, and reproduction of the orientation of the crossbar, a significant difference is expected between the two categories of orientations for each task: orthogonal and diagonal. From the previous experiment there are solid grounds to expect such a difference in the two water-level tasks. It is expected that a similar difference between the diagonal and orthogonal orientations is to be found in the balance task as well.

With respect to the effect of age on the solution of the tasks, it is predicted that there will not be a significant difference between age groups (age of subjects ranges from 6 years to 9 years) as far as the orthogonal positions of the three apparatus are concerned, a difference is expected in the responses of the age groups for the diagonal positions of the each of the three apparatus.

The second type of hypotheses regard the order of acquisition of the different tasks and task conditions. A number of hypotheses can be made as to the order of acquisition. I will divide the hypotheses in three categories. The first contains the order of acquisition for the two containers of the water-level problem, the second contains the predictions as to the two conditions with the balance and finally the third category of hypotheses regards the acquisition orders holding between the water-level task in its various conditions and the balance task.

The first set of hypotheses is a the replication of the results found in Experiment 1.

Hypothesis 1a:

There is a collective *décalage* between the reproduction of the water-line when the square container is oriented orthogonally and when it is oriented diagonally. Orthogonal orientations are systematically solved earlier in development than diagonal orientations.

Correct performance at the diagonal orientation should be more strongly associated with correct performance at the orthogonal than with incorrect performance at the orthogonal orientations.

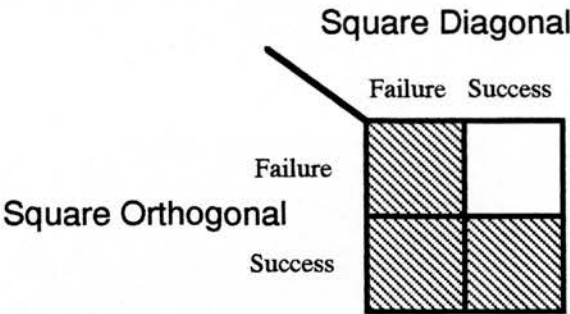


Figure 6.1 Model of collective *décalage* between responses to the diagonal and orthogonal orientations of the square bottle according to Hypothesis 1a (the white cell is predicted to be empty).

Hypothesis 1b:

There is a collective *décalage* between the reproduction of the water-line when the round container is oriented orthogonally and when it is oriented diagonally. Orthogonal orientations are systematically solved earlier in development than diagonal orientations.

Correct performance at the diagonal orientation should be more strongly associated with correct performance at the orthogonal than with incorrect performance at the orthogonal orientations.

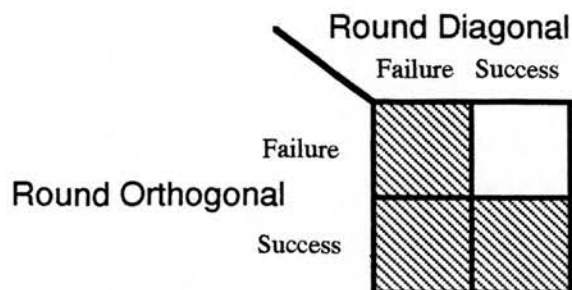


Figure 6.2 Model of collective décalage between responses to the diagonal and orthogonal orientations of the round bottle according to Hypothesis 1b (the white cell is predicted to be empty).

Hypothesis 1c:

The reproduction of the water-level in the condition in which the square container is oriented orthogonally is concurrent with the reproduction of the water level when the round container is oriented orthogonally.

Correct performance on the orthogonal square should be strongly associated with correct performance on the orthogonal round; incorrect performance on the orthogonal square should be strongly associated with incorrect performance on the orthogonal round. Subjects will either fail both tasks or succeed in both.

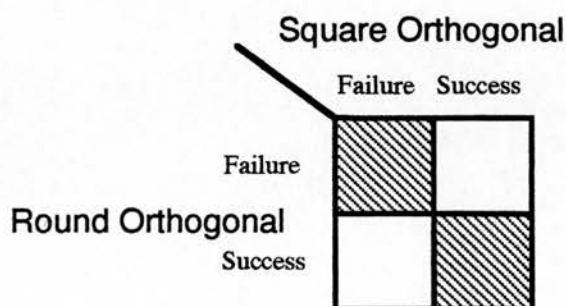


Figure 6.3 Model of concurrence between responses to the orthogonal orientations of the square bottle and orthogonal orientations to the round bottle according to Hypothesis 1c (the white cells are predicted to be empty).

Hypothesis 1d:

There is a collective décalage between the reproduction of the water-line when the round container is oriented diagonally and the reproduction of the water-level when the square bottle is oriented diagonally. Diagonal orientations of the round container are systematically solved earlier in development than diagonal orientations of the square

container. Correct performance on the diagonal square should be more strongly associated with correct performance on the diagonal round than with incorrect performance on the diagonal round.

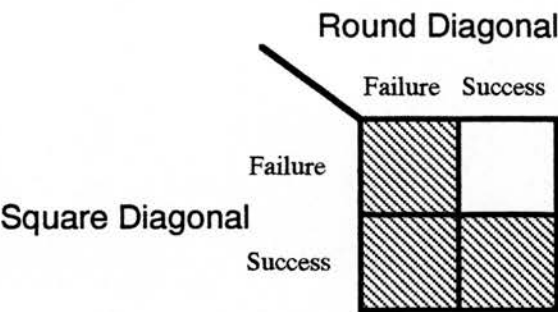


Figure 6.4 Model of collective décalage between responses to the diagonal orientations of the round bottle and the diagonal orientations of the square bottle, according to Hypothesis 1d (the white cell is predicted to be empty).

The second type of hypotheses regards the acquisition order holding between the two conditions with the balance, the orthogonal position and the diagonal position.

Hypothesis 2:

There is a collective décalage between the reproduction of the orientation of the crossbar for the orthogonal position of the balance, and for the diagonal position of the balance.

Correct performance at the diagonal balance should be more strongly associated with correct performance at the orthogonal balance than with incorrect performance at the orthogonal balance.

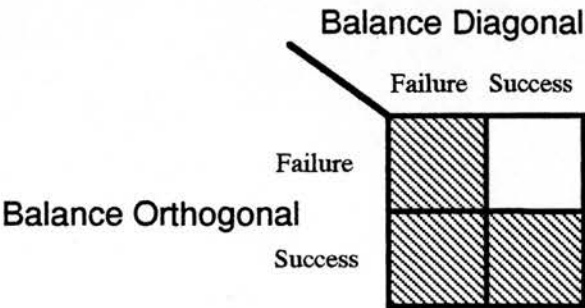


Figure 6.5 Model of collective décalage between responses to the diagonal and orthogonal orientations of the balance according to Hypothesis 2 (the white cell is predicted to be empty).

The third set of hypotheses regard the order relation holding between the responses at the balance task and the responses in the water-level task. A comparison between the various tasks in their respective conditions gives rise to a series of predictions as to the order with which the various conditions will be solved.

Hypothesis 3a:

The reproduction of the water-level in the condition in which the square container is oriented orthogonally is concurrent with the reproduction of the orientation of the crossbar when the balance is oriented orthogonally.

Correct performance on the orthogonal square should be strongly associated with correct performance on the orthogonal balance; incorrect performance on the orthogonal square should be strongly associated with incorrect performance on the orthogonal balance. Subjects will either fail both tasks or succeed in both.

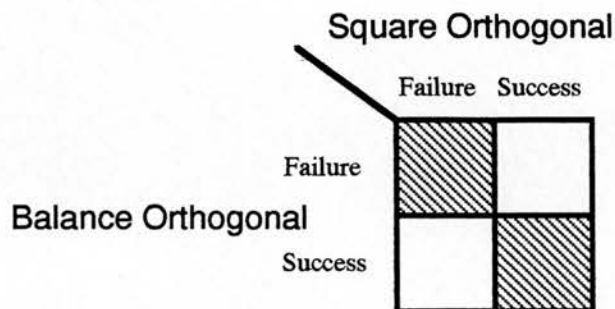


Figure 6.6 Model of concurrence between responses to the orthogonal orientations of the square bottle and orthogonal orientations of the balance according to Hypothesis 3a (the white cells are predicted to be empty).

Hypothesis 3b:

The reproduction of the water-level in the condition in which the round container is oriented orthogonally is concurrent with the reproduction of the orientation of the crossbar when the balance is oriented orthogonally.

Correct performance on the orthogonal round should be strongly associated with correct performance on the orthogonal balance; incorrect performance on the orthogonal round should be strongly associated with incorrect performance on the orthogonal balance. Subjects will either fail both tasks or succeed in both.

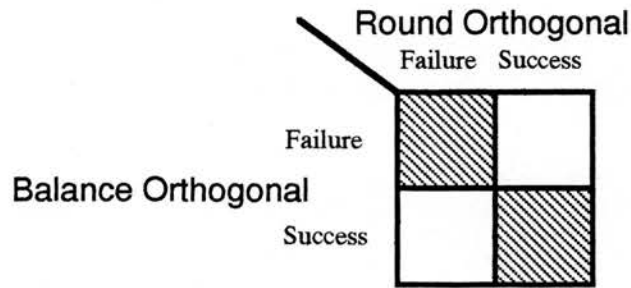


Figure 6.7 Model of concurrence between responses to the orthogonal orientations of the round bottle and orthogonal orientations to the balance according to Hypothesis 3b (the white cells are predicted to be empty).

Hypothesis 3c:

There is a collective *décalage* between the reproduction of the orientation of the crossbar of the balance in the diagonal position, and the reproduction of the water-level in the diagonal position of the square bottle.

Correct performance in the diagonal square should be more strongly associated with correct performance at the diagonal balance than with incorrect performance at the diagonal balance.

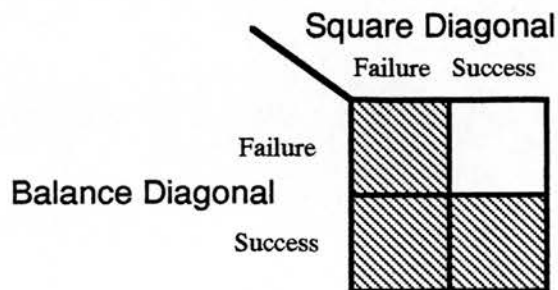


Figure 6.8 Model of collective *décalage* between responses to the diagonal of the square bottle and diagonal orientations of the balance according to hypotheses 3c (the white cell is predicted to be empty).

Hypothesis 3d:

There is a collective *décalage* between the reproduction of the orientation of the crossbar of the balance in the diagonal position, and for the diagonal position of the round bottle.

Correct performance in the diagonal round should be more strongly associated with correct performance at the diagonal balance than with incorrect performance at the diagonal balance.

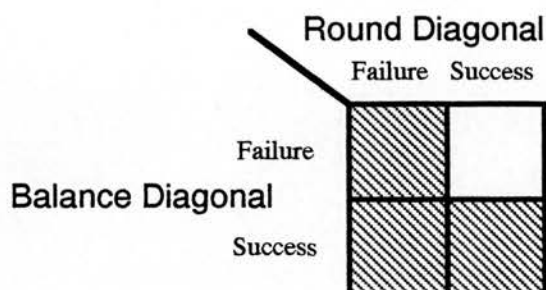


Figure 6.9 Model of collective *décalage* between responses to the diagonal of the round bottle and diagonal orientations of the balance according to hypothesis 3d (the white cell is predicted to be empty).

6.4 METHOD

6.4.1 Design

There were three tasks: reproduction of the water-level in the square bottle, reproduction of the water-level in the round container, reproduction of the crossbar in the balance task. The order of presentation of the three tasks was randomised. Subjects could start with the square bottle then pass to the balance task and then draw the water-level for the round bottle, or begin with the balance then pass to the two water-level tasks, etc..

The design of the two water-level conditions is equivalent to the design of Experiment 1. There are 8 possible angles of inclination of each container. The angles are the following: the 4 orthogonal 0° , 90° , 180° , 270° ; and the 4 diagonal 45° , 135° , 225° , 315° . Each subject will draw 8 lines representing the water-level. 4 lines for the round bottle and 4 lines for the square. Each subject sees each bottle at 4 angles, 2 orthogonal and 2 diagonal. Therefore each subject sees each bottle in only half of the possible orientations. This is done in order to keep the number of drawings per child to 8. There are therefore 2 groups of subjects in each class, seeing a different set of

orientations. Results from the previous study showed that there was no effect of group, we therefore assume that the two sets of angles are equivalent.

The balance task introduced two conditions: an orthogonal condition when the balance was presented with the pivot upright perpendicular to the base, a diagonal condition when the pivot was tilted at 45 degrees with respect to the base. Children were always asked to do the orthogonal setting before the diagonal setting. Two settings for each condition were required. Therefore there were two settings with the balance upright and two settings with the pivot of the balance tilted at 45 degrees. One setting was with the pivot at 45 degrees to the right and one setting with the pivot at 45 degrees to the left.

The following *independent variables* are manipulated:

- physical complexity of the task:
 - water-level task in its two conditions: round and square bottle
 - balance problem
- task condition or angle of orientation:
 - orthogonal
 - diagonal
- schooling/age group:
 - primary 2 (between 6 and 7 years)
 - primary 3 (between 7 and 8 years)
 - primary 4 (between 8 and 9 years)

The *dependent variable* is the number of correct reproductions of the orientation of the water-level and of the crossbar.

6.4.2 Statistical analysis

Two types of statistical treatments have been used in this study, the first to analyse results between subjects and the second to verify the hypotheses of order of acquisition.

For the first level analysis two nonparametric tests were employed in view of the fact that responses were measured as number of subjects failing and succeeding the tasks¹⁰ :

- the McNemar test for significance of changes in related groups was employed to evaluate differences between the orthogonal and diagonal conditions for each of the three tasks;
- χ^2 test was employed to evaluate the differences between age groups' distribution of correct responses in each of the three tasks for the two conditions orthogonal and diagonal;

For the second level of analysis I tested the hypotheses of order of task acquisition, using Hildebrand, Laing & Rosenthal (see Section 3.5). The Prediction Analysis of Cross-Classifications method offers a means to evaluate which hypothesis of order of acquisition of two tasks better fits the data. The 4 possible models are: concurrence, décalage in favour of Task1, décalage in favour of Task2, no systematic order between tasks. Extended presentation of this method is found in Chapter 3.

For each hypothesis of order between two tasks, the predictions have been tested by computing a Del index which gives the improvement over chance after one or two cells supposed empty. The contingency tables present data based on allocating children in the two response categories: pass or fail.

6.4.3 Apparatus and Procedure

The experimental equipment consists in 3 apparatus for the 3 tasks. The first two elements are the same as in the previous Experiment1 and consist of the glass sphere and the square bottle employed in the previous study.

¹⁰ Contrary to Experiment 1 between subjects analysis is carried out by nonparametric tests. As will be explained in the section on response measure, the type of measures taken for the Balance task did not allow a precise quantification of the angle of tilt of children's reproduction of the Crossbar. Their answers were simply categorised as correct or wrong depending on the fact that they had set the "sticker" representing the crossbar at an angle superior or inferior to 10 degrees off the horizontal. It follows, that comparison between the Balance and the water-level tasks has been carried out on the basis of the number of correct and wrong responses even though children's line drawings of the water-level could be measured in angles as in Experiment 1.

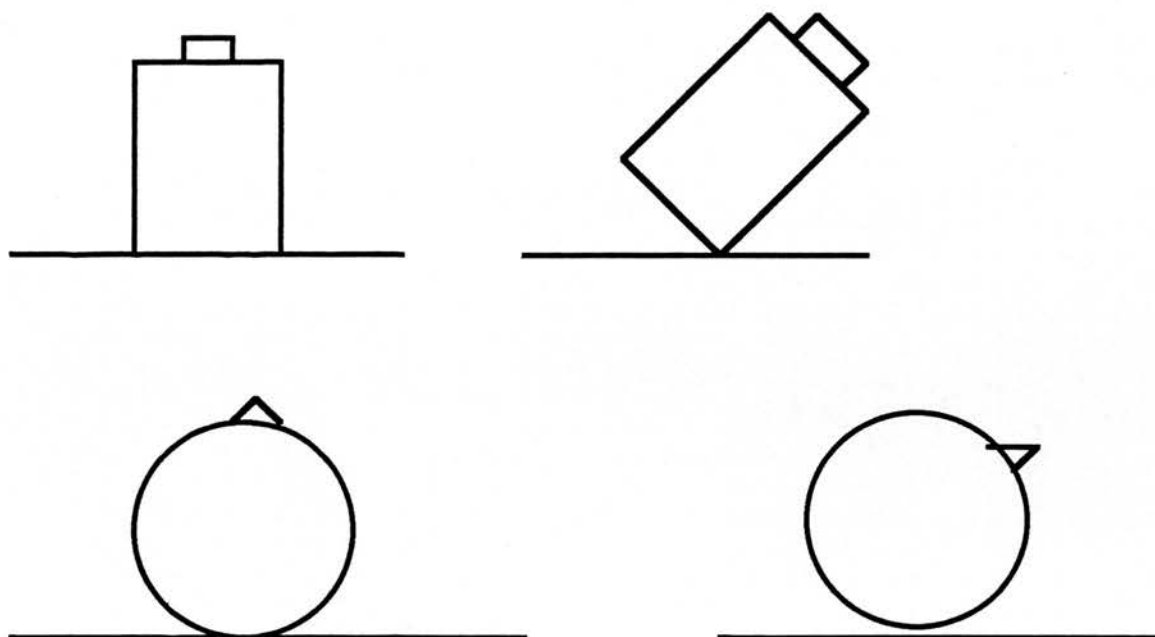
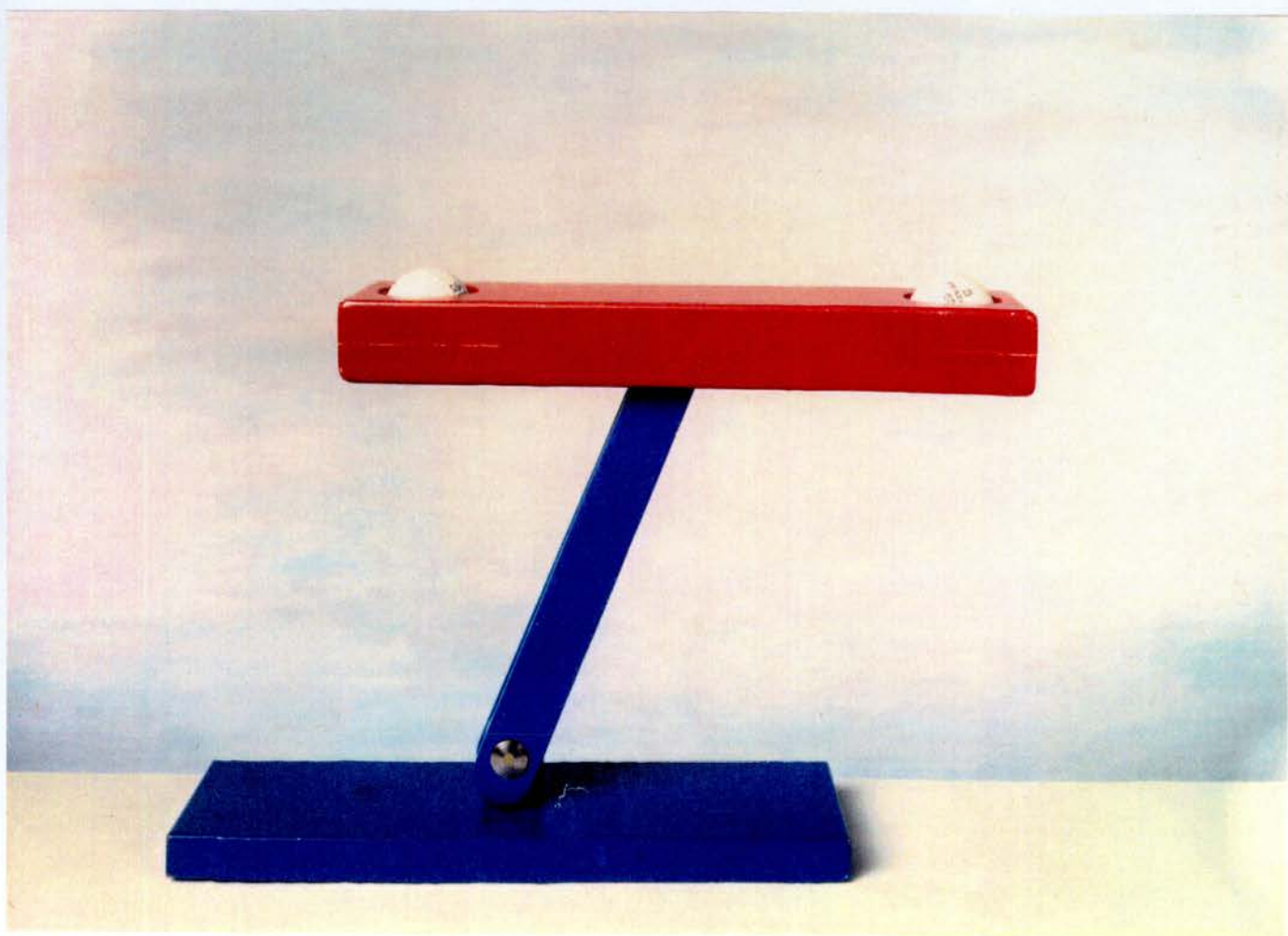


Figure 6.10 The square and round bottles

The procedure employed for the two water-line conditions with the two containers, is an exact replication of the procedure used in Experiment 1. As in the previous experiment, children were asked to draw the level of the water on a card where was depicted the side outline of the bottle. On each card there was one bottle tilted to a certain angle, corresponding to the angle to which the real bottle was positioned. The child is given a red pencil to draw with. The bottle remains visible and tilted at the corresponding angle while the child is drawing. The question is repeated for the 8 tilts of the bottles. For supplementary details see Chapter 6.

Situation 3 employed a simplified version of a balance scale. The balance was a wooden object composed of a base, a bar or pivot, and an crossbar which could move on a pivot point. The crossbar of the balance had an indentation at each extremity in which a small object could be placed, these "holes" replacing the usual plates. The side outline of the crossbar was therefore a simple rectangle. The base and pivot were painted blue, the crossbar red (photographs next page).

When two objects placed at each end of the crossbar are of equal weight the crossbar remains horizontal, on the contrary if the weights are different the crossbar pivots on its center and tilts on the heaviest side.



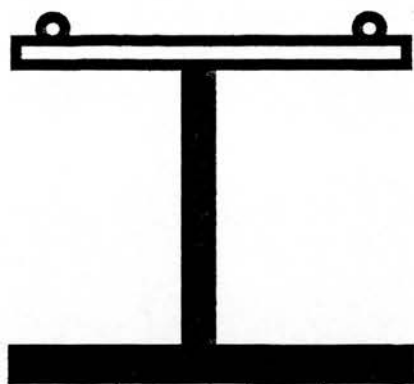


Figure 6.11 The balance in the upright orthogonal position

A feature of this balance is that the pivot is itself mobile and can be inclined to the side at an angle of ± 45 degrees with respect to the base. The crossbar moves independently from the support and hence, with equal weights in the holes the crossbar remains horizontal regardless of the inclination of the pivot.

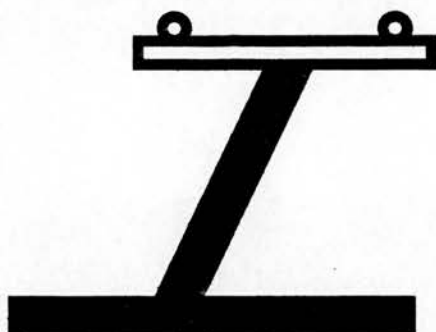


Figure 6.12 The balance in the tilted diagonal position

The procedure was the following: I initially assessed children's understanding of the working of the balance. The child first explored the object and moved all the elements of the balance. Subject were shown how the pivot of the balance could move and tilt in different directions. A few little balls of different weight were then presented. Two balls were golf balls and two were coloured plastic balls. The difference in weight was extremely significant and perfectly recognisable by picking them up and holding them in a hand. The balance was put in its canonical position with the pivot perpendicular to the base. A subject was asked to predict what would happen when two balls of the same weight were put in the two holes at the extremity of the crossbar. The prediction was tested by inserting the two golf balls on the balance. Then the child was asked to

predict what would happen if on one side was put a golf ball and on the other a plastic ball. The prediction was again tested by putting the two different balls on the balance. Once I estimated that the subject's understanding and knowledge of the working of the balance was adequate, the actual experiment started. Children who did not exhibit knowledge of the functioning of a balance and made incorrect predictions (e.g. children that could not predict that with two equivalent balls the bar would stay "level" or children who predicted that in the case of unequal balls, the side that would "go down" was the one on which the lighter ball was placed) were excluded from the experiment. Only two subjects failed the pretest.

Children were then presented with a prepared card on which a side drawing of the base and crossbar had been drawn. They were also given a rectangle of red cardboard which represented the crossbar of the balance. The balance was put in position and then subjects were asked to place the "sticker" on the drawing (it is sticky on one side and adheres to the cardboard of the card) in order to reproduce the position of the crossbar of the balance.

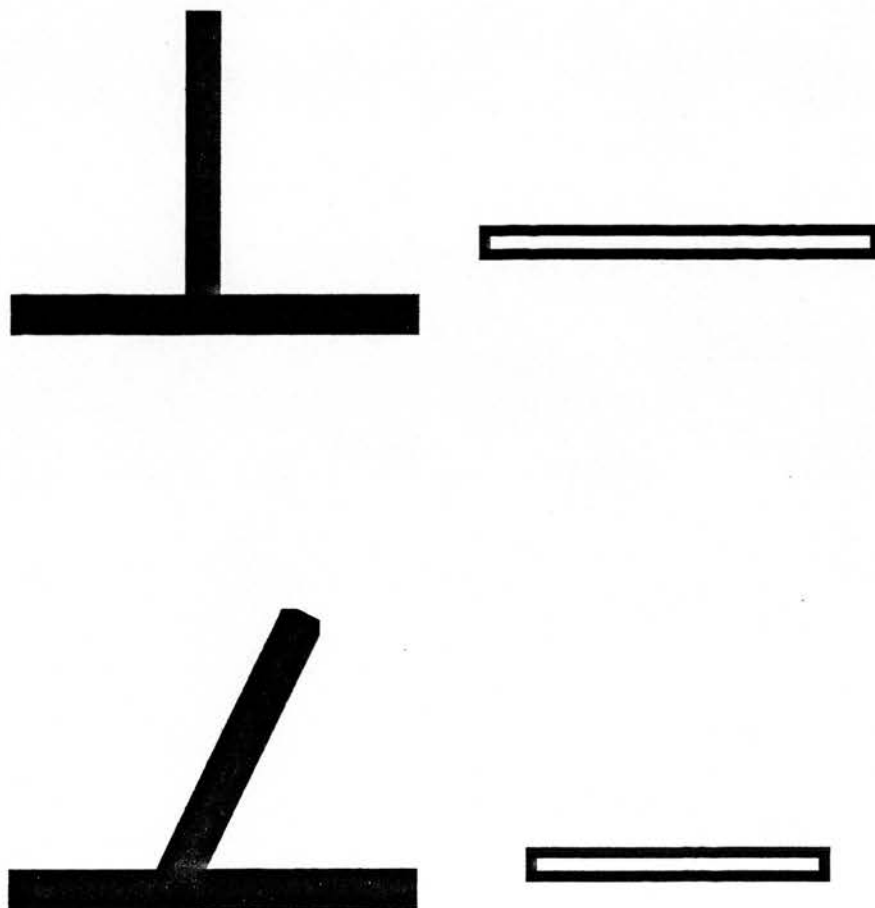


Figure 6.13 The side outline of the base and pivot of the balance and the "sticker" representing the crossbar.

There were 4 prepared cards: 2 showing the balance in a orthogonal position i.e. with the pivot upright and perpendicular to the base; and two diagonal positions of the balance i.e. with the pivot tilted at 45 degrees, once 45° to the right and once 45° to the left. The cards were presented one at the time and children could set and reset the sticker various times until they were satisfied. The balance was put in one of the orientations before the child was given the card and remained visible all the time.

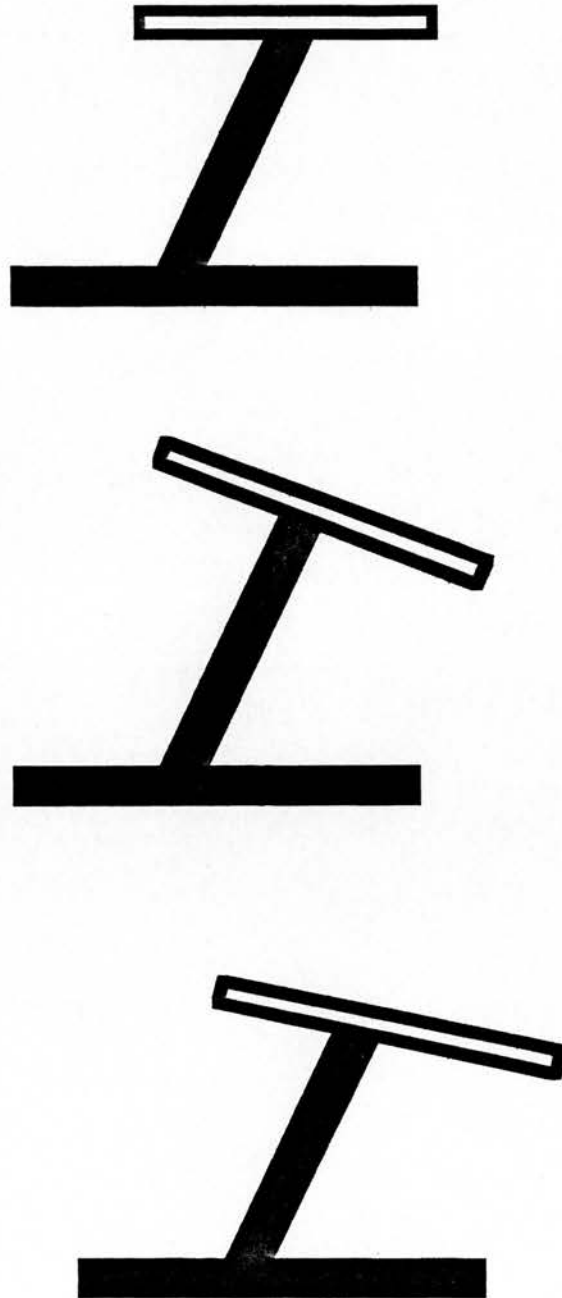


Figure 6.14 Some possible correct and incorrect settings of the "sticker" on the diagonal outline of the balance.

When the rectangle was set correctly it was oriented horizontally. Incorrect settings either consisted in the rectangle being set perpendicular to the pivot or simply at an incorrect angle:

6.4.4 Interviews

The interviews occurred in one of the rooms of the school. Children were seen individually for a session of 15/20 minutes. The experimenter sat in front of the child and moved the various pieces of equipment and asked the questions. The experimenter also noted the results of children's settings for the balance task and took notes of any particular behaviour or interesting remark uttered by the child.

6.4.5 Subjects

Subjects were 60 children from a school in central Edinburgh. Age ranges from 6 years to 9 years, classes primary 2, primary 3 and primary 4. There are 20 subjects from each class. Mean age for primary 2 was 6.8 years ($SD = .23$); mean age for primary 3 was 7.6 years ($SD = .24$) and mean age for primary 4 was 8.7 years ($SD = .26$).

6.4.5 Measure

For the two water level tasks with round and square bottles, responses were measured as angles off the horizontal. Measures of the subject's drawings were taken with a protractor. The orientations of the lines produced could span from 0 to 180 degrees. When lines were not drawn and children produced scribbles or coloured the totality of the figure, the drawings were coded as errors of 180 degrees. Wavy lines were approximated to the best fit, and this average line was measured with the protractor. Any line tilted 10 degrees or less was considered correct. The 10 degrees allowing for drawing errors. Over 10 degrees off 0° the water line was considered incorrectly oriented. In the contingency tables, where subjects were classified as succeeding or failing, the criterion for failure was that a subject had drawn one or more incorrect water-levels.

For Task 3 the balance, responses were measured as angles off the horizontal. The angle formed by the pivot and the rectangular "sticker" was taken with a protractor.

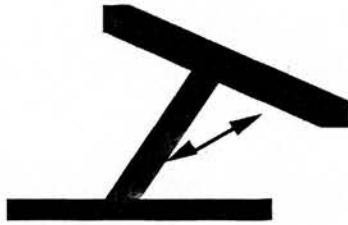


Figure 6.15 The angle between the crossbar and pivot of the balance measured with the protractor.

The particular experimental situation did not allow measures as precise as the ones in the water-level task. The outlines of the balance on which the "stickers" were set and the stickers themselves were used repeatedly for all the subjects. It was therefore not possible to preserve each child's response for further scrutiny, as with the drawings of the water-level. The measures of the 4 settings of the sticker on the 4 outlines had to be carried out at the end of each interview. For this reason the 4 settings were measured solely to control whether the angle was over 10 degrees and then categorised as correct or wrong. Any setting tilted 10 degrees or less was considered correct. Over 10 degrees off the horizontal the orientation was considered incorrect.

In the contingency tables, where subjects were classified as successful or failing in each of the two conditions, the criterion for failure was that a subject had done one or more incorrect setting for each condition.

6.5 Results

The analysis of the results is performed in three phases. The group performance analysis examines the effect of the two conditions orthogonal diagonal in each task and the effect of the age/class variable on the performance in each condition of the three tasks. Finally in the third phase a hierarchical analysis examines the order of acquisition of the tasks.

6.5.1 First level analysis: comparison of the orthogonal and diagonal conditions

The first analysis of the results was conducted to verify our hypotheses on the difference between the two conditions diagonal/orthogonal within each task.

		Diagonal	
		Fail	Succ
Ortho	Succ	39	15
	Fail	6	0

Square Bottle

		Diagonal	
		Fail	Succ
Ortho	Succ	15	39
	Fail	5	1

Round Bottle

		Diagonal	
		Fail	Succ
Ortho	Succ	27	33
	Fail	0	0

Balance

Table 6.1 Correct and failed orientations for the orthogonal and diagonal conditions of the three tasks: square bottle, round bottle and balance.

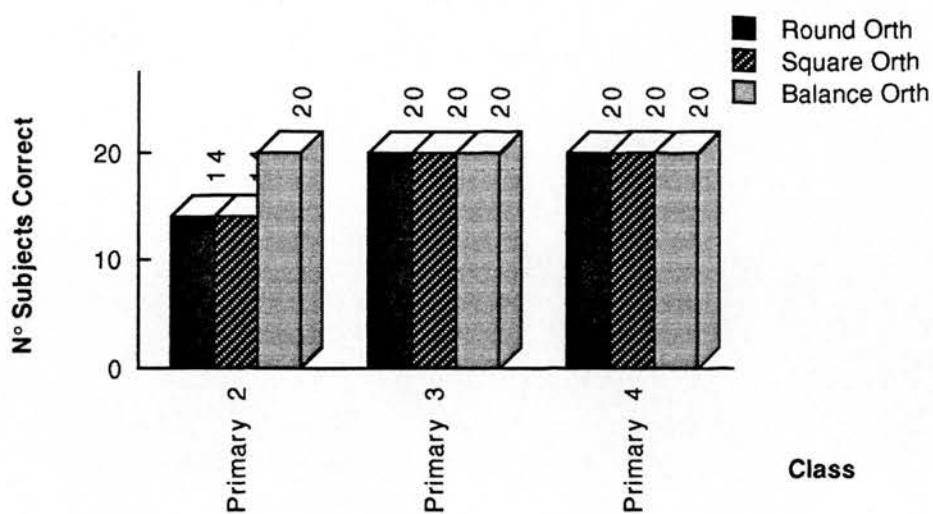
Inspection of these tables indicates that the frequencies of correct and incorrect subjects varied considerably between the orthogonal and diagonal conditions for each task. In each case it appears that the orthogonal condition elicited significantly more correct responses than the diagonal condition. Specifically, the McNemar test for significance of changes in related groups indicated that in every case there was a significant difference between condition orthogonal and condition diagonal.

In the square bottle task, the difference in responses between orthogonal and diagonal was significant ($\chi^2 = 37.02$ df.=1 $p < .001$). In the round bottle task, the difference between the orthogonal and diagonal condition was also significant ($\chi^2 = 10.5$ df.=1 $p < .01$). In the balance task, there were significantly more correct responses in the orthogonal than the diagonal condition ($\chi^2 = 25.03$ df.=1 $p < .001$).

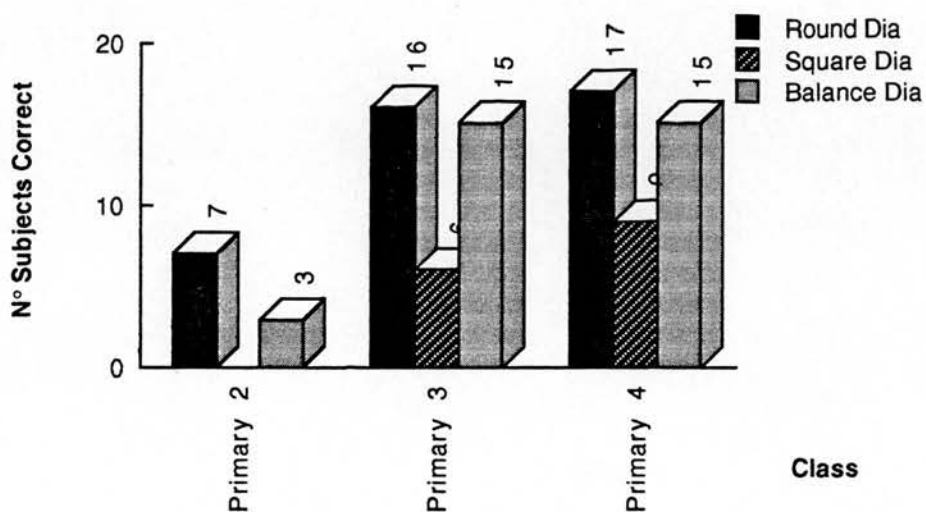
In conclusion for the three tasks we can reject the null hypothesis that there is no change in subjects' responses to the two conditions: orthogonal and diagonal. In all the tasks, there is a significant change in the number of correct responses between the orthogonal and diagonal conditions. It also appears from the tables that the orthogonal condition in every case elicits a greater number of correct responses.

6.5.2 Age group comparison

A second analysis was carried out to determine the age period in which some developmental change occurs on the basis of the difference in age groups' performance.



Graph 6.1 Number of subjects in each class performing correctly in each of the three tasks in the orthogonal condition.



Graph 6.2 Number of subjects in each class performing correctly in each of the three tasks in the diagonal condition.

The following tables, 6.2, 6.3, 6.4, present the contingency tables of the children's correct and wrong responses to each task condition as a function of age. A χ^2 test is computed on each contingency table. Occasionally a further breakdown of the table is required to identify the precise age groups in which the performance varies significantly.

Task 1, the Water-Line task with the square bottle showed a significant improvement with age in the number of correct responses, for the diagonal condition but not for the orthogonal condition.

**Square Bottle
Orthogonal**

age Res	P2	P3	P4
Succ.	14	20	20
Fail	6	0	0

**Square Bottle
Diagonal**

age Res	P2	P3	P4
Succ.	0	6	9
Fail	20	14	11

Table 6.2 Correct and failed orientations by age group in the two conditions orthogonal and diagonal for the square bottle

Because of the ceiling effect in the orthogonal condition, with zero scores in 2 cells, it was not possible to compute a valid χ^2 . Even if we consider only the two groups Primary 2 and Primary 3 the distribution is still not significantly different from chance distribution. The binomial test is not significant $p=.196$

Condition square diagonal: $\chi^2 (2, N=60) = 11.12$ $p < 0.05$ The χ^2 test is significant and indicates that there is a significant difference from chance in the distribution of responses across age groups in the diagonal condition. The difference is essentially due to the distribution of the responses in Primary 2. In fact, when Primary 2 and Primary 3 responses are compared, a significant difference is found: $\chi^2 (1, N=40) = 6.34$ $p < 0.02$ On the other hand the difference between response distribution in Primary 3 and Primary 4 is not significant: $\chi^2=0.96$ $p > 0.30$

For Task 2, the Water-Line task with the round bottle, results in the orthogonal positions are identical to the square bottle in the same orthogonal orientations. However there is a different distribution of responses in the diagonal condition.

Round Bottle Orthogonal				Round Bottle Diagonal			
age Res	P2	P3	P4	age Res	P2	P3	P4
Succ.	14	20	20	Succ.	7	16	17
Fail	6	0	0	Fail	13	4	3

Table 6.3 Correct and failed orientations by age group in the two conditions orthogonal and diagonal for the round bottle

Condition round orthogonal: as in the case of the orthogonal square condition, the ceiling effect with zero scores in two cells did not allow to compute a valid χ^2 . The comparison of Primary 2 to Primary 3 did not yield a significant result, the binomial test was not significant $p = .196$

Condition round diagonal: $\chi^2 (2, N=60) = 13.86 p < 0.001$ The χ^2 test is significant. The difference is essentially due to the distribution of the responses in Primary 2. In fact, when Primary 2 and Primary 3 responses are compared, a significant difference in response distribution is found: $\chi^2 (1, N=40) = 8.1 p > 0.0$ On the other hand the difference between response distribution in Primary 3 and Primary 4 is not significant: $\chi^2 = 0.09 (1, N= 40) p > 0.70$

In the balance task, no difference between the 3 age groups was found, in the orthogonal condition as all of subjects in each class were successful in orienting the crossbar. Differences are found however in the solution of the diagonal condition.

		Balance Orthogonal		
age	Res	P2	P3	P4
	Succ.	20	20	20
	Fail	0	0	0

		Balance Diagonal		
age	Res	P2	P3	P4
	Succ.	3	15	15
	Fail	17	5	5

Table 6.4 Correct and failed orientations by age group in the two conditions orthogonal and diagonal for the balance

Condition balance diagonal : $\chi^2 (2, N=60) = 19.2, p < 0.001$ The distribution of correct and failed responses in the three groups is significantly different from the chance distribution. There is a significant increase in correct responses with age. In particular, when we compare P2 and P3 the difference is significant $\chi^2 (1, N=40) = 14.4 p < 0.001$ There is no difference in the number of correct and wrong responses of Primary 3 and Primary 4.

In conclusion significant changes with age are observed in all the tasks for the diagonal conditions, while no significant improvement in number of correct responses with age appears in any of the orthogonal conditions. The developmental differences emerge essentially between Primary 2 and the other two age groups which tend not to differ between each other.

6.5.3 Prediction analysis of acquisition orderings

The testing of the specific developmental hypotheses of concurrency and décalage in the solution of the 4 conditions of the water-level task, and the 2 conditions of the balance task is based on Hildebrand, Laing and Rosenthal's Prediction Analysis of Cross Classifications.

Hypothesis 1a: collective décalage between orthogonal and diagonal orientations for the square bottle.

The hypothesis was made that there is a collective décalage between orthogonal and diagonal orientations of the square bottle, in favour of the orthogonal. The triangular hypothesis associated with collective décalage in favour of orthogonal solutions over diagonal solutions predicts that the cell corresponding to failure in the orthogonal and success at the diagonal be empty. In the language of prediction analysis we predict that

Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models.

The table below indicates the number of subjects having failed or succeeded in the orthogonal and diagonal positions for the square container.

		Square Diagonal	
		Failure	Success
Square Orthogonal	Failure	6	0
	Success	39	15

Table 6.5: Contingency table of Success and Failure in the orientation of the water-level for orthogonal and diagonal Orientations of the square container

Mod 3: Del = 1 $z = e$, $p < 0.00001$

$z_{M1-M3} = -28.2$ $p < 0.00003$

$z_{M2-M3} = -54.3$ $p < 0.00003$

Model 3, décalage in favour of Task1, orthogonal square has proved significant. The Del is equal to 1 and the model is a significantly better predictor than than any of the other two models: concurrence or the opposite collective décalage. This result indicates that when children solve correctly the orthogonal condition they can either fail or succeed in the diagonal condition. Instead if they fail the orthogonal they systematically fail the diagonal as well.

Hypothesis 1b: collective décalage between orthogonal and diagonal orientations for the round bottle.

We made the hypothesis that there is a collective décalage in the solution of the orthogonal and diagonal orientations of the round bottle, in favour of the orthogonal. The triangular hypothesis associated with collective décalage in favour of orthogonal solutions over diagonal solutions predicts that the cell corresponding to failure in the orthogonal and success at the diagonal be empty. In the language of prediction

analysis we predict that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models.

		Round	
		Diagonal	
Round Orthogonal	Failure	5	1
	Success	15	39

Table 6.6: Contingency table of success and failure to orthogonal and diagonal Orientations for round container

Mod 3: Del = .75 $z = 3.3$ $p = 0.00048$

$z_{M1-M3} = -2.6$ $p = 0.0046$

$z_{M2-M3} = -2.8$ $p = 0.0025$

Model 3, décalage in favour of Task1 i.e.orthogonal round has proved significant. The model is a significantly better predictor than than any of the other two models: concurrence or the opposite collective décalage. This result indicates that when children solve correctly the orthogonal condition they can either fail or succeed in the diagonal condition, however if they fail the orthogonal they systematically fail the diagonal as well.

Hypothesis 1c: concurrence between orthogonal orientations for square and orthogonal orientations for round containers.

The hypothesis that orthogonal angles are solved concurrently in both containers predicts that Model 2 concurrence between task 1 and task 2 will be significant and will be significantly better than the other 2 models. The ability to orient the water-level correctly in the orthogonal square condition appears concurrently with the ability to orient correctly the water-level in the orthogonal round condition . Therefore subjects will either solve both tasks or fail both tasks. The table below indicates the number of subjects having failed or succeeded in the orthogonal positions for the square and round containers.

		Square Orthogonal.	
		Failure	Success
Round Orthogonal	Failure	4	2
	Success	2	52

Table 6.7 Contingency table of success and failure to orthogonal orientations for the square and the round container

Mod1 = .629 ($z = 3.6$ $p < 0.003$)

Mod2 = .629 ($z = 3.6$ $p < 0.003$)

Mod3 = .629 ($z = 3.6$ $p < 0.003$)

$z_{M1-M2} = 0$

$z_{M1-M3} = 0$

$z_{M2-M3} = 0$

The fact that all three models are significant without there being a significantly different from each other, indicates that there is a weak concurrence between the two conditions. This is because all the models predict at the same time that the two bottom left hand and top right hand cells be empty. The weak concurrence results as opposed to strict concurrence is due to the extremely unbalanced distribution: 54 subjects out of 60 (90% of subjects) succeed both tasks.

Hypothesis 1d: collective décalage between diagonal orientations for the square bottle and diagonal orientations of the round bottle.

We made the hypothesis that there is a collective décalage between diagonal orientations of the square bottle and diagonal orientations of the round bottle in favour of the diagonal round. The triangular hypothesis associated with collective décalage in favour of round solutions over square solutions predicts that the cell corresponding to failure in the round diagonal and success at the square diagonal be empty and that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models.

The table below indicates the number of subjects having failed or succeeded in the diagonal positions for the square and round containers.

		Square Diagonal	
		Failure	Success
Round Diagonal.	Failure	20	0
	Success	25	15

Table 6.8: Contingency table of Success and Failure in the orientation of the water-level in the conditions diagonal square and diagonal round.

Mod 3: $\Delta = 1$; $z = e p < 0.0001$

$z_{M1-M3} = -4.8$ $p < 0.00003$

$z_{M2-M3} = -15.9$ $p < 0.0003$

Model 3, *décalage* in favour of Task1, diagonal round has proved significant. The Δ is equal to 1 and the model is a significantly better predictor than any of the other two models, concurrence or the opposite collective *décalage*. This result indicates that when children solve correctly the diagonal round condition, they can either fail or succeed in the diagonal square condition, however if they fail the diagonal square, they systematically fail the diagonal round as well.

Hypothesis 2: collective *décalage* between orthogonal and diagonal orientations of the balance.

We made the hypothesis that there is a collective *décalage* in the solution of the orthogonal and diagonal orientations of balance, in favour of the orthogonal. The triangular hypothesis associated with collective *décalage* in favour of orthogonal solutions over diagonal solutions predicts that the cell corresponding to failure in the balance diagonal and success at the orthogonal balance be empty and that Model 3, *décalage* in favour of Task1 will be significant and significantly better than the other 2 models. The table below indicates the number of subjects having failed or succeeded the orthogonal and diagonal positions for the balance.

		Balance Diagonal	
		Failure	Success
Balance Orthogonal	Failure	0	0
	Success	27	33

Table 6.9 Contingency table of correct and failed reproductions of the orientation of the crossbar in the conditions balance orthogonal and balance diagonal.

From this table it appears that 100% of the subjects succeed in the orthogonal condition, however only 55% also succeed in the diagonal condition. With this distribution, none of the developmental hypotheses tested is significant (the Del indexes are negative), this does not allow us to draw any conclusions on the acquisition order between the two tasks. The problem may be due to the sampling, as none of the children fail both tasks and all the subjects succeed the balance orthogonal condition. It does not really tell us about the stage leading to the success in the orthogonal condition. However what must be stressed, is that two competence levels emerge: one in which subjects are successful in the orthogonal condition while still failing the diagonal condition, and one in which they succeed in both. There are thus, *no* subjects that succeed the diagonal condition while failing the orthogonal, a result which is compatible with our initial hypothesis and which confirms the order relation holding between the two conditions.

Hypothesis 3a: concurrence between square orthogonal and orthogonal balance.

The hypothesis was made that the reproduction of the orientation of the crossbar when the balance is orthogonal concurs with the reproduction of the orientation of the water-level when the square container is orthogonal. This hypothesis consists in claiming that the ability to orient the water-level correctly in the orthogonal square condition appears concurrently with the ability to orient correctly the crossbar in the orthogonal balance condition. Children are expected to either fail both tasks or succeed in both. The table below indicates the number of subjects having failed or succeeded the orthogonal and positions for the balance and for the round container.

		Balance Orthogonal.	
		Failure	Success
Square Orthogonal	Failure	0	6
	Success	0	54

Table 6.10 Contingency table of correct and failed reproductions of the orientation of the crossbar in the conditions balance orthogonal and of the water-level in the condition square orthogonal.

This table is again very unbalanced. The majority of subjects (90%) succeed in both tasks and no subject fails the balance task nor fail both tasks. Here again the test of the triangular hypotheses does not make much sense: the Dels are all negative. What the table does show, is that within the age range examined almost all the subjects succeed in the two conditions and that success in the balance task precedes slightly success in the square bottle. This data supports the interpretation that there is a level in which subjects are capable of representing the orientation of the horizontal plane for the two tasks in the orthogonal condition. In order to examine the acquisition process leading to this competence and to identify more precisely the eventual orders of acquisition of the two tasks, a younger age group should be examined.

Hypothesis 3b: concurrence between round orthogonal and orthogonal orientation of the balance.

The hypothesis was made that the ability to reproduce the orientation of the crossbar when the balance is orthogonal concurs with the ability to reproduce the orientation of the water-level when the round container is orthogonal. This hypothesis predicts that the ability to orient the water-level correctly in the orthogonal round condition appears concurrently with the ability to orient correctly the crossbar in the orthogonal balance condition. Children are expected to either fail both tasks or succeed in both. The table below indicates the number of subjects having failed or succeeded the orthogonal and positions for the balance and for the round container.

		Balance Orthogonal	
		Failure	Success
Round Orthogonal	Failure	0	6
	Success	0	54

Table 6.11 Contingency table of correct and failed reproductions of the orientation of the crossbar in the conditions balance orthogonal and of the water-line in the condition round orthogonal.

These results are the exact replica of the results found in the previous table where the responses to the balance orthogonal and square orthogonal tasks are presented. Therefore the same conclusions can be drawn that this experiment identifies a competence level corresponding to correct orientations to the orthogonal orientations of the balance and round bottle.

Hypothesis 3c: collective décalage between balance diagonal and square diagonal.

We made the hypothesis that there exists a collective décalage between the correct reproduction of the orientation of the crossbar in the condition in which the balance is at diagonal orientations and later the correct reproduction of the water-level for the diagonal orientations of the square bottle. No subjects are expected to solve the diagonal square bottle while failing the diagonal balance. In the language of prediction analysis we predict that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models.

		Square Diagonal	
		Failure	Success
Balance Diagonal	Failure	27	0
	Success	18	15

Table 6.12 Contingency table of correct and failed responses to the water-level problem in the diagonal condition of the square bottle and diagonal condition of the balance.

Mod 3: Del = 1; $z = e$, $p < 0.00001$

$z \text{ Mod1-Mod3} = -6.1$, $p < 0.00003$

$z \text{ Mod2-Mod3} = -9.62$, $p < 0.00003$

Model 3, collective *décalage* in favour of the balance diagonal is a significantly better predictor than the other models. The child's performance in the balance diagonal is a reliable predictor of performance in the diagonal square. When subjects solve correctly the diagonal balance they can either succeed or fail in the diagonal square, however if they fail the diagonal balance they systematically fail the diagonal square.

Hypothesis 3d: collective *décalage* between balance diagonal and round bottle diagonal.

We made the hypothesis that there is a collective *décalage* between the correct reproduction of the orientation of the crossbar in the condition in which the balance is at diagonal orientations and the correct reproduction of the water-level for the diagonal orientations of the round bottle. The *décalage* is expected to be in favour of the diagonal balance: no subject will solve diagonal round while failing the diagonal round. In the language of prediction analysis we predict that Model 3, *décalage* in favour of Task1 will be significant and significantly better than the other 2 models. The table below indicates the number of subjects having failed or succeeded the diagonal positions for the square and round containers.

		Round Diagonal	
		Failure	Success
Balance Diagonal.	Failure	15	12
	Success	6	27

Table 6.13 Contingency table of correct and failed responses to the diagonal condition of the balance and the water-level problem in the diagonal condition of the round bottle.

Mod1: Del = .38 $z 3.1$, $p < 0.00097$

Mod 2: Del = .31 $z 2.8$, $p < 0.0025$

Mod 3: $\Delta_1 = .48$ $z = 3.0$, $p < 0.00135$

As all three Δ s are significant. At the same time, the difference between them is not statistically significant: z values of comparisons between the models are of the order of 0.25 $p = 0.40$. We can only conclude that there is a weak concurrence between the two tasks as the prediction that the bottom left hand cell and the top right hand cell are empty, is confirmed. Contrary to our initial hypothesis it seems that the solution to the diagonal conditions of the balance and round container emerge in a related way close to concurrence.

6.6 Discussion of the results

The result which emerges more clearly from this experiment is the difference in number of correct responses between the two conditions orthogonal and diagonal in the three tasks. The "orthogonal effect", already reported in Experiment 1, appears to be a general developmental phenomenon that goes beyond the water-level problem. The results with the balance have confirmed the role of the orientation of the display in children's representation of the horizontality of a plane.

As far as the age group comparison are concerned, the results have revealed that within our sample there is a ceiling effect in the orthogonal conditions as practically all the subjects succeed. However a developmental process is observed in the age range examined, as there is a significant improvement with age in the diagonal conditions. The age group of the subjects had been chosen on the basis of results of Experiment 1, which covered a large age range going from 5 to 12 years. In that experiment, it had appeared that Primary 1 subjects still had a few drawing difficulties and occasionally reverted to scribbles. For this reason, in this experiment children after Primary 1 from 7 to 10 years were chosen, they also constituted the most interesting age to observe the transition from failure to success in the diagonal conditions. I consider that focusing on this age group has been a good choice to concentrate on the shift from orthogonal to diagonal conditions amongst subjects who could solve the orthogonal conditions.

The acquisition orders between orthogonal and diagonal conditions give particularly clear results for the diagonal conditions in all three tasks. I will discuss the orders identified, one by one and then attempt a more general discussion of the relation between tasks.

6.6.1 The water-level task

A conclusion that can be drawn from this study is that the results of Experiment 1 have been replicated. The acquisition orders found in the previous experiment on the water-level problem have been confirmed in this second experiment. The "orthogonal effect" in which the water-level in the orthogonal orientations of the container is correctly represented before the diagonal orientations of the container, has been replicated for both shapes of bottles. As far as the orders holding between the two containers, the orthogonal conditions have again appeared to be solved concurrently, while a collective *décalage* has been observed once again between the round and square bottles in favour of the round container.

6.6.2 The balance task

The results from the balance task, have confirmed the effect of the task condition: orthogonal vs diagonal. The conditions are significantly different and are acquired in order; while none of the subjects failed in the orthogonal condition 50% did not succeed in reproducing the horizontal orientation of the crossbar in the diagonal condition. It is not surprising that all of the children were correct in reproducing the correct orientation of the crossbar in the upright position of the pivot as all the subjects were "competent" as to the working of the balance and were good predictors of the position the bar should assume when equal weights are put on the scales. In the pretest it appears clearly that the children could use the horizontal orientation of the crossbar as an index of the equivalence in the weight of the two golf-balls. Therefore their good understanding of the functioning of the balance seems to lead to a correct representation and reproduction of the position of the crossbar on the bidimensional space of the card.

The difference in results when the pivot is oriented diagonally brings additional support to the view that there is a stage in which children organise their spatial relations on the basis of the features internal to the object. From the pretest and the results in the orthogonal condition we know that subjects have a clear understanding that when the weights are equal the crossbar stays horizontal. However when the pivot is at 45 degrees these same subjects, reproduce the crossbar tilted at an angle that can be either perpendicular to the pivot or simply at an angle of 20 or 30 degrees. In other words these subjects are not capable of reproducing the horizontal orientation of the crossbar

when the crossbar is *not* perpendicular to the pivot bar (recall that the balance is visible in front of them).

Let us reexamine the two task conditions. In the orthogonal condition the situation is such that the crossbar is not only perpendicular to the direction of gravity but it is also perpendicular to the pivot. On the contrary, when the pivot is oriented to 45 degrees the coincidence between the direction of gravity and the vertical axis of the balance is broken (as in the diagonal condition of the water-level task). In this case the horizontal crossbar is perpendicular to the direction of gravity but at an angle with the pivot. In this situation children fail to reproduce the horizontality of the crossbar.

These results are surprisingly similar to those in the water-level task; there seems to be a stage in which children can only represent the horizontality of the plane when the plane is perpendicular both to gravity and to the axes of the object of which it is part and by which it is supported. When there is no such correspondence the child cannot represent the orientation of the plane. The question is, whether the problem the child faces is one of not being able to rely solely on the relation of the plane with an external referent such as the direction of gravity because such an abstract referent is not available to him, or whether he can only recognise the horizontality when the internal and external referents coincide.

The evidence we have gathered suggests that the latter interpretation is the most probable. In fact as was discussed in the conclusions to the previous experiment, it appears that children attribute spatial features to objects in equilibrium such as a top a bottom and some internal axes, relying on the vertical and horizontal axes of the environment and specifically on the direction of gravity. This referent is relevant to construct the spatial features of canonical and noncanonical objects, features which we have seen determine an "internal object space" to which other elements such as the water are referred. The direction of gravity or the vertical as Olson & Bialystock have argued, does seem to be available to young children as a reference system to attribute spatial features to objects. Therefore it would appear once again, that it is not the lack of this referent that generates the errors with the diagonal conditions of our tasks but rather the mismatch of the direction of gravity with the axes or elements of the supporting objects.

At a later stage children come to consider the orientation of the crossbar, which is dynamically independent from the rest of the balance, as relative solely to the direction of gravity and to other planes in the environment. The correspondence of "internal" and "external" vertical axes ceases to be necessary to determine the horizontality of the plane.

6.6.3 The acquisition orders between the water-level tasks and the balance task

The results in the three tasks have shown that there is a stage in which the orthogonal conditions are solved successfully and concurrently. The vast majority of subjects has solved the three orthogonal conditions successfully. While this result does not allow us to say anything about the process by which the competence to represent the water-level horizontally in the orthogonal conditions (for this a younger sample of children of which some may fail the task, should be examined), it does suggest that from age 5, children are capable of orienting a plane horizontally when the supporting contiguous elements are oriented orthogonally. This competence seems to be sufficiently general to be found in tasks involving radically different physical contexts, like the balance and the water tasks.

The clear homogeneity we find in the orthogonal conditions is not reproduced in the diagonal conditions of the tasks. There are two orders which emerge from the study. A net anticipation of the balance diagonal with respect to the square bottle in the diagonal position, and a weak concurrence of the balance and the round bottle. This result proposes again the question of the relation holding between the round and square bottle versions of the water-level task. The data from the balance task, may provide a better explanation of the *décalage* between the two conditions of the water-level problem that we had left unsolved in the previous experiment.

The question can be approached from two distinct angles: why is the performance in the square bottle so markedly different from the other task conditions? What are the reasons that allow a quasi synchronous solution of the round bottle task and the balance task? We may consider that subjects that have succeeded in the diagonal conditions of the round and balance tasks, have reached a mature or "top level" representation and that therefore the explanation of the late solution of the square diagonal condition may pertain to specific features of this task. On the contrary, the failure at the square diagonal, may be the index that the concept of horizontality, when the other two diagonal conditions are solved successfully, is only partially elaborated. The first hypothesis seems to be the most likely given our results.

Although the concurrence between the diagonal orientation of the balance and of the round container is only of the weak type, it may be claimed that there is a transition from the "orthogonal stage" of the object centered space, to the "diagonal stage" of the capacity of establishing the horizontal orientation of a plane regardless of the orientation of the supporting objects. The square container is acquired at an even later stage because it may introduce further requirements. The water-level inside the square

bottle, has the particularity of appearing oblique, there is a perceptual bias towards seeing it tilted to some degree. In order to overcome this effect it may be necessary to know that the water-level stays horizontal. It seems necessary to impose the physical principle of the invariance of the liquid surface on the misleading perceptual appearance in order to correctly represent the orientation of the level. It may be the case that with the round container this knowledge is not an a priori necessity, because the round bottle does not introduce this perceptual bias. The horizontal can be discovered during the experiment and applied to each new orientation of the bottle.

Children have often commented during the experiment that the water stays "level" in the round bottle but not in the square. When asked why, they usually respond that in the sphere the water stays level because the "ball is round and the water can go everywhere", while the "square bottle has corners so the water goes in the wee corners and stays squint". Therefore while accepting that the level is horizontal in the sphere they do not consider that it is the same in the square bottle because the water-level looks "squint".

The horizontal invariance of the water-level is only partially acknowledged and depends on the perception of the water-line which itself varies with the configuration of the container. What is particularly surprising is that the arguments that justify the invariance of the level in the round bottle are the same that will be subsequently used also for the square bottle. In fact children invoke more or less indirectly a principle of gravity: they say that "the water must stay flat because it has to go down" or "that it cannot stay up it always falls down and fills everything equally". When asked why this is not the case in the square bottle they say that the water "goes a little bit squint because the bottle is squint and it has to go down inside the corners".

In conclusion I suggest that the *décalage* of the diagonal square is to be interpreted as a special case in the development of the horizontal. The specific configuration of the apparatus requires subjects to impose their knowledge of the invariance of liquid's orientation on the perceptual appearance of the water-level. The physical knowledge plays here a fundamental role in the correct reproduction of the horizontal surface of the water. I would consider therefore, that subjects who succeed only with the round diagonal have not acquired a permanent abstract principle of the invariance orientation of liquid surfaces, but are only capable of recognising it in some conditions (with the sphere, with puddles etc). These subjects are capable of invoking some of the physical principles that determine the position of the water-level but these principles cannot override the perceptual bias that makes the water look tilted in the square jar. This limitation in their physical knowledge is not to be taken, therefore, as a limitation in their spatial knowledge, which on the contrary seems to have progressed considerably

from the previous stage (as appears from the concurrent solution at the balance diagonal condition).

6.6.4 The general hierarchy

The experiment had set out to investigate whether a different physical context allowed children to abandon the "object centered" representation of the horizontality of a plane before the water-level problem was successfully solved. The results have shown that this is the case if we take success at the square diagonal to be the criterion for correct reproduction of the water-level. If on the contrary, we take success at the round diagonal to be the significant criterion for the acquisition of the water-level principle a concurrence between the two physical contexts has emerged. The latter effect together with the concurrence that has been found in the solution of all the orthogonal conditions brings new insight to children's spatial conceptions during this period of development, a problem which constituted the second objective of this study.

The progress constituted by the correct reproduction of the orientation of the crossbar and of the water-level in the diagonal conditions can be interpreted as the acquisition of a new competence in relating these planes to the framework offered by the vertical direction of gravity and its perpendicular. While in the previous stage the plane was recognised and reproduced as horizontal only when there was a correspondence between the vertical axes of the object and the direction of gravity or when the perpendicularity to the internal axis corresponded to the perpendicularity to the vertical in the environment, in this stage the plane can be oriented in isolation independently from the relations to its contiguous elements. Horizontality is no longer tied to the relation of the plane with its supporting elements.

But how is one to interpret this dependence on the internal features of the object? I would argue that the evidence of the balance, where subjects had a good understanding of the dynamics of the crossbar and the role of the weights, suggests that during a first stage children require that a horizontal plane be "doubly horizontal": horizontal with respect to the environment and horizontal with respect to the internal axes of the object. Horizontality is *realized* as a perpendicular both to the direction of gravity as a perpendicular to the vertical axis of the object. When no such correspondence exists, the child does not represent the horizontality of the plane. He stresses instead the angle (over or below 90 degrees) that holds between the parts of the object. Later on the contrary, the dynamic independence of the plane determines that the child look for its independent orientation. The plane is treated as an independent object that must be

oriented separately from its contiguous elements by disregarding the relations that tie it to the rest of the object.

An open question remains: how do children come to represent the horizontal plane correctly in the orthogonal conditions? As we have seen from the age of 5 children are correct in these conditions for all three tasks; very little can be said on the order of acquisition of these problems and of the type of difficulties they may encounter at a younger age. Furthermore it is not clear if the orthogonal effect is the output of a process of development or is present from a very early age. Until now there has been limited investigation in children's competence in problems of spatial reference before the age of 5 because of the difficulties young children experience in representing graphically surfaces such as the water level. The next experiment will focus on a younger age group and propose an experimental situation that is more suited for 3 and 4 years old subjects.

Chapter 7: Experiment 3

7.1 Introduction

In this last experiment I focus on the 4 and 5 years age group, a period that has been slightly overlooked in the literature mainly because of young children's inability to draw straight lines and thus to produce significant results at the water level task. From my previous experiments it has emerged that there is a clear stage in which children are capable of representing the horizontal orientation of a plane when the supporting elements of the plane are orthogonal to the environment. This has appeared with the water-level problem as with the balance problem. A question one may want to ask is what precedes this stage. In Experiment 2, the youngest subjects I interviewed were 6 years old. From age 6 therefore, the vast majority of children are successful in the orthogonal conditions of the tasks. We have little if no evidence on the developmental process leading to this stage and on the behaviours that precede it. This Experiment studies the performance of a younger sample of subjects in the two tasks investigated in previous experiments. Replicating the water-level and balance tasks with a younger group of children may inform us as to the type of responses and more important, the order of acquisition of the orthogonal conditions of the balance and water-level tasks.

It is however very likely, that the younger subjects may fail the two tasks because of the complexity of the tasks themselves, i.e. the difficulty in drawing, a bad comprehension of the dynamic properties of the objects involved or simply a poor comprehension of the task requirements. These difficulties may therefore mask the spatial competence we are attempting to assess, and prevent us from investigating nursery children's abilities in orienting planes and using reference systems. In fact, if nursery children were to represent the water-level with a scribble and be unable to set the sticker in any significant orientation (e.g. sticking it on the base or on the side rather than on the outlined pivot) we would not be in the position of saying very much on their competence to orient planes and to make use of the system of coordinates¹¹.

A new task has been therefore designed that is targeted to our younger subjects. In particular this new situation addresses the question of whether given a simple practical

¹¹ We will test nursery children with the water-level task notwithstanding these difficulties mainly for methodological reasons. As was explained in chapter 3, the idea behind a battery of tasks is to compare the performance of each subject in the range of horizontality tasks in order to assess in which situations they are successful and in which they fail and thus establish the extension of their understanding and the range of the conceptual theory that is held at various stages of development. I predict that the water-level task will be failed by nursery children and I wish to show that the failure in this task is concurrent with success in other structurally similar tasks of horizontality. The water-level task will thus be one of the tasks of the battery.

situation young children have the ability of orienting a surface horizontally. Or in other words, whether there are contexts in which the task of orienting a plane horizontally is a meaningful objective for young children and which they have the means to achieve. In fact before dismissing children's (before the age of 5) performance in tasks of spatial reference, as unmeasurable or as the index of children's inability to understand the issue of spatial orientation, situations suited to their age must be attempted.

The new situation I wish to introduce has been designed to probe young children's competence in orienting a plane horizontally. The orientation task has been adapted to young children in that it does not require subjects to reproduce the orientation of the plane with graphic means such as drawing or graphic representations as the sticker. Children will be asked instead to directly orient the object itself. The interest of a practical task has been supported by the results collected in a pilot study with Nursery children. In fact, the difficulty in drawing straight lines to represent the water-level, reported by Piaget was confirmed. The pilot study also indicated that although Nursery children could easily place the sticker and thus had no problems with the response means of the balance task, they had a very limited understanding of the working of the balance itself.

The situation I designed involves a simple wooden apparatus which I have called the Table. The table has two legs, a base and a table top. The legs of the table can be shortened or lengthened by pushing or pulling them out of the base. The legs can be lengthened independently and therefore one leg can be longer than the other, which causes the table top to be tilted with respect to the base. The table can also be inserted on an inclined base or wedge. When the table is on the wedge and the legs of the table are the same length the table top is tilted. In order for the table top to be horizontal when the base is on the wedge, one of the legs must be lengthened in order to counteract the inclination of the base. The apparatus is colourful and easy to manipulate and globally quite appealing to 3 and 4 years old children.

With this apparatus the question emerged as how to ask a young child to set a plane such as the table top horizontally; what is the appropriate means to convey the idea of this spatial orientation? The solution I found takes advantage of children's knowledge of the dynamics of objects and of their equilibrium. The task I asked the subjects to perform is to set the table top in such a way as to keep a little ball in equilibrium. All the children knew very well that if the table top is tilted a ball put on its top would roll off, therefore could understand the request of putting the table top "so that the ball would stay still and not fall". At the same time the fact that the ball stays in equilibrium when the table top is horizontal is a measure and a test of the horizontality of the plane.

No additional measures must be taken to assess that the subjects has set the table correctly.

As in previous tasks, I introduced two conditions: the orthogonal and diagonal conditions. This is to evaluate whether children's reliance on the "object centered" features that we have identified in the two previous tasks, characterises children's responses to the new task in this early stage of development as well. It may be the case on the contrary, that with a practical task the "orthogonal effect" disappears. Children may show that in these contexts they not only can orient a plane horizontally but can also rely on a framework centered system of coordinates.

In the orthogonal condition, the table is on a horizontal base and in order to obtain a horizontal top the two legs must have the same length. Therefore children start with a situation in which the legs are different length and have to produce a symmetric configuration. The legs must be made equal with the table top perpendicular to the two legs and parallel to the base. This is a canonical configuration for a table and the horizontal plane is both perpendicular to the direction of gravity and perpendicular to its supporting legs. The internal and external axes correspond and there is no conflict between them.

In the diagonal condition, the table is on the wedge and to obtain a horizontal table top the legs must be different lengths, the top is not perpendicular to the legs, and the configuration is globally asymmetric. Furthermore the vertical axis of the table given by the direction of the legs, does not correspond to the direction of gravity. The table top has a horizontal orientation which is independent from the orientation of the rest of the table. The internal and external axes do not coincide. In this situation in order to orient the table top horizontally subjects must refer the table top to an external frame, the direction of gravity or another horizontal plane in the environment.

The two conditions respect the task requirements that were introduced in the other tasks. In fact as in the previous experiments, the table task involves a plane which must be oriented horizontally, a plane which is dynamically independent from the other elements which support it. As in the original water-level task, a transformation in the spatial relations holding between the object and the environment must entail a transformation in the spatial relations holding between the plane and the other elements of the object. Differently from the other two tasks the table top does not "find its horizontal orientation" by necessity and independently, but must be actively put in the horizontal position by the subject himself. However as in the previous tasks the horizontality of the plane is a physical necessity in the sense that the table top in its role of support of the ball must be horizontal to preserve a state of equilibrium.

A final comment should be made on the consequences of employing a practical task which does not require subjects to represent or reproduce the orientation but to practically orient the plane itself. Although a difference exists between the tasks in the response mode, I will treat the tasks as 3 comparable tasks all evaluating the child's capacity to *represent* a horizontal plane. The table task in fact, requires the child to anticipate a certain spatial state of the table top, and to evaluate this state before testing it with the small ball. In this sense, while not requiring to answer the problem with a symbolic representation on paper, it requires anticipation and mental representation of the spatial relation that is here being investigated¹².

7.2 Objectives

The first objective is to assess children's behaviours in the stage preceding the success at the orthogonal conditions of the two tasks employed in previous experiments: the water-level task and the balance task. In particular, the question is the order of acquisition of the two tasks. The results of the previous experiments in which all the subjects succeeded in the various orthogonal conditions did not allow us to determine whether the water-level and balance orthogonal conditions were acquired concurrently or in order. By examining a younger group of subjects this question may find an answer. The age range has been chosen as to observe the transition from failure to success in these conditions: from Nursery to Primary 2. In fact in accordance with previous results, it is expected that Primary 2 subjects succeed in the orthogonal conditions of the two tasks. Behaviours of subjects preceding that class remain to be explored.

The second objective of the study is to evaluate children's competence in orienting a plane horizontally with a practical task. Establishing the acquisition order between the orthogonal and diagonal conditions of the table task will provide further evidence on the transition from an object to a framework centered representation of orientations. It may be the case that with a practical task children more easily take into consideration the orientation of the plane with respect to the environment, therefore the two conditions will be solved concurrently. On the contrary it may be that there is a collective *décalage* between the orthogonal and diagonal conditions as is the case in the other tasks. In other words we would observe the same transition from relying on both internal and external features to determine the orientation of the table top to reliance on external features uniquely.

¹² The interpretation of the results will rely on the assumption that the three tasks are equally representational and are thus comparable.

The third objective is to establish the order holding between the three tasks. In particular whether the table task will be solved successfully in its two conditions before the other two tasks. It may be the case that all the orthogonals are solved concurrently and all before the success at any of the diagonal conditions of the tasks. This result would support a hypothesis of a very general stage in which "object centered" properties are represented. On the contrary it is possible that the table task follows its own course of development and that therefore it precedes, in its two conditions, the successful solution of the water-level and balance task. This result would introduce the issue of the response means and the difference between tasks which are practical and tasks which require a symbolic reproduction by the transposition of the orientation of the plane onto a bidimensional space of the page.

Overall the experiment should provide additional data on the conditions in which children are able to orient a plane horizontally and conversely the conditions in which children fail in doing so and on this basis allow us to identify different competence levels. This will allow us to refine our characterisation of the transformation of the concept of horizontality in the period of development that goes from age 4 to age 10.

7.3 Hypotheses

The first type of hypotheses regard the differences between task conditions. For the three tasks, water-level reproduction with the round bottle, reproduction of the orientation of the crossbar, and orientation of the table, a significant difference is expected between the two categories of orientations: orthogonal and diagonal. From the previous experiment there are solid grounds to expect such a difference in the water-level tasks and the balance task. It is expected that a similar difference between the diagonal and orthogonal orientations is to be found in the table task as well.

With respect to the effect of age on the performance in the tasks, it is predicted that there will not be a significant difference between the three age groups as far as the orthogonal positions of the table is concerned, however it is expected that Nursery children will give more incorrect responses to the balance and water-level problem. In fact the water-level problem requires children to draw the line which is a competence young children may not possess as has been shown in the literature. As far as the balance task is concerned, it is possible that many Nursery children be uncertain as to the working of the apparatus. A difference is also expected in the responses of the three age groups for the diagonal positions of the each of the three apparatus.

The second type of hypotheses regards the order of acquisition of the different tasks and task conditions. A number of hypotheses can be made as to the order of acquisition. I will divide the hypotheses in three categories the first deals with the order of acquisition for the two containers of the water-level problem, the second consists of predictions as to the two conditions with the balance and finally the third category of hypotheses regards the acquisition orders holding between the water-level task in its various conditions and the balance task. Some of these hypotheses derive from the results in previous experiments, others and in particular the ones regarding the new table task are derived from the results of a pilot study conducted in a nursery school.

Hypothesis 1a:

There is a collective *décalage* between the reproduction of the water-line when the round bottle is oriented orthogonally and when it is oriented diagonally. Orthogonal orientations are systematically solved earlier in development than diagonal orientations.

Correct performance at the diagonal orientation should be more strongly associated with correct performance at the orthogonal than with incorrect performance at the orthogonal orientations. Thus, if we consider a contingency table with responses for orthogonal orientations and responses for diagonal orientations of the round bottle, the cell corresponding to success at the diagonal and failure at the orthogonal, is predicted to be empty.

		Bottle Diagonal	
		Failure	Success
Bottle Orthogonal	Failure		
	Success		

Figure 7.1 Model of collective *décalage* between responses to the diagonal and orthogonal orientations of the round bottle according to Hypothesis 1b.

Hypothesis 1b:

There is a collective *décalage* between the reproduction of the orientation of the crossbar of the balance for the orthogonal position of the scale, and for the diagonal position of the balance.

Correct performance at the diagonal balance is more strongly associated with correct performance at the orthogonal balance than with incorrect performance at the orthogonal balance.

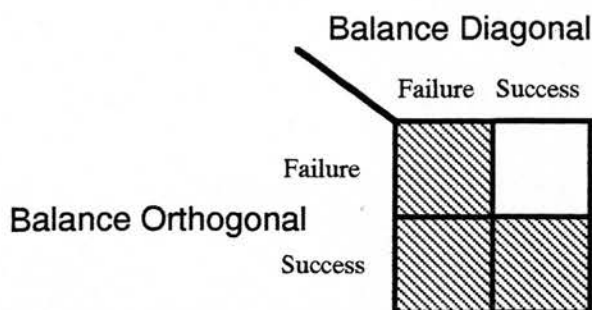


Figure 7.2 Model of collective décalage between responses to the diagonal and orthogonal orientations of the balance according to Hypothesis 1b (white cell is predicted to be empty).

Hypothesis 1c:

There is a collective décalage between the placement of the table top when the base is horizontal (orthogonal condition) and when the base is inclined, i.e. when the table is placed on the wedge (diagonal condition). The orthogonal condition is systematically solved earlier in development than the diagonal condition.

Correct performance at the diagonal condition should be more strongly associated with correct performance at the orthogonal condition than with incorrect performance at the orthogonal condition.

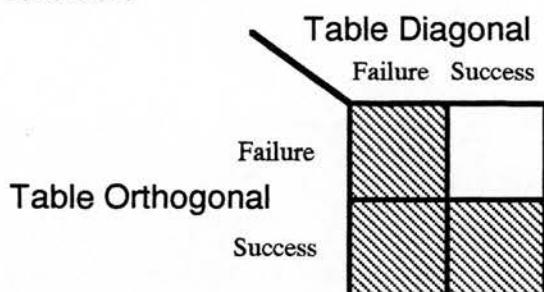


Figure 7.3 Model of collective décalage between responses to the diagonal and orthogonal orientations of the table according to Hypothesis 1c (white cell is predicted to be empty).

The second set of hypotheses regard the order relation holding between the correct responses at the balance task and the correct responses in the water-level task and the correct responses at the table task. A comparison between the various conditions and orientations gives rise to a series of predictions as to the order with which the various conditions will be solved.

Hypothesis 2a:

The reproduction of the water-level in the condition in which the bottle is oriented orthogonally is concurrent with the reproduction of the crossbar when the balance is oriented orthogonally.

Correct performance on the orthogonal bottle should be strongly associated with correct performance on the orthogonal balance; incorrect performance on the orthogonal bottle should be strongly associated with incorrect performance on the orthogonal balance. Subjects will either fail both tasks or succeed in both.

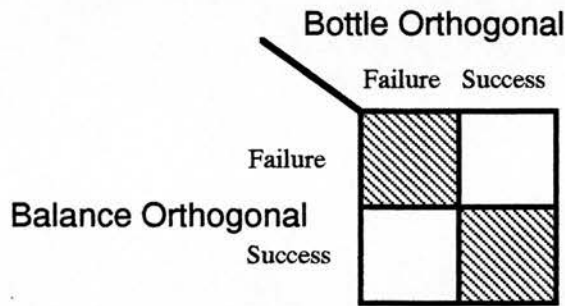


Figure 7.4 Model of concurrence between responses to the orthogonal bottle and orthogonal orientations to the balance according to Hypothesis 2a (white cells are predicted to be empty).

Hypothesis 2b:

There is a collective *décalage* between the reproduction of the orientation of the crossbar of the balance in the diagonal position, and for the diagonal position of the bottle. Correct performance in the diagonal bottle should be more strongly associated with correct performance at the diagonal balance than with incorrect performance at the diagonal balance.

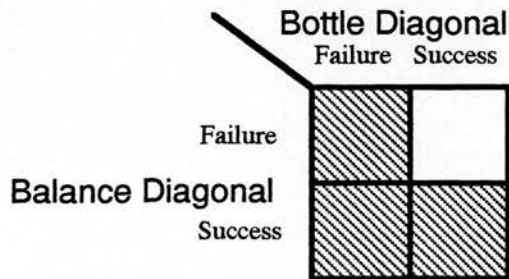


Figure 7.5 Model of collective *décalage* between responses to the diagonal of the bottle and diagonal orientations of the balance according to hypothesis 3d (white cell is predicted to be empty).

Hypothesis 2c:

There is a collective *décalage* between the reproduction of the orientation of the water level in the orthogonal position of the round bottle and the orientation of the table top in the orthogonal position.

Correct performance in the orthogonal bottle should be more strongly associated with correct performance at the orthogonal table than with incorrect performance at the orthogonal table.

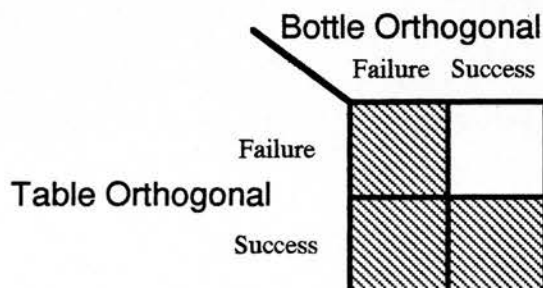


Figure 7.6 Model of collective *décalage* between responses to the orthogonal bottle and orthogonal orientations of the table according to hypothesis 2c (white cell is predicted to be empty).

Hypothesis 2d:

There is a collective *décalage* between the reproduction of the orientation of the water level in the diagonal position of the round bottle and the orientation of the table top in the diagonal position.

Correct performance in the diagonal Round bottle should be more strongly associated with correct performance at the diagonal table than with incorrect performance at the diagonal table.

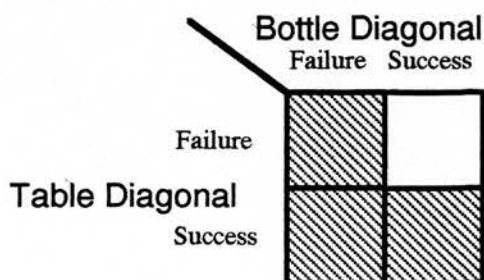


Figure 7.7 Model of collective *décalage* between responses to the diagonal bottle and diagonal orientations of the table according to hypothesis 2d (white cell is predicted to be empty).

Hypothesis 2e:

There is a collective décalage between the reproduction of the orientation of crossbar in the orthogonal position of the balance and the orientation of the table top in the orthogonal position.

Correct performance in the orthogonal balance should be more strongly associated with correct performance at the orthogonal table than with incorrect performance at the orthogonal table.

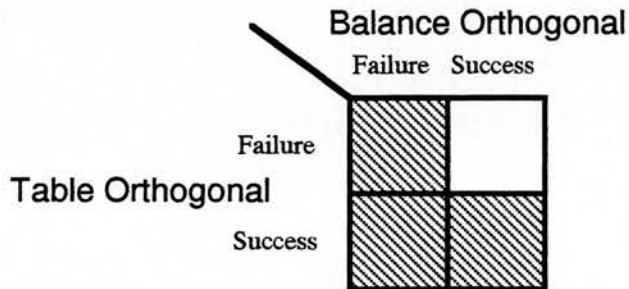


Figure 7.8 Model of collective décalage between responses to the orthogonal balance and orthogonal condition of the table according to hypothesis 2e (white cell is predicted to be empty).

Hypothesis 2f:

There is a collective décalage between the reproduction of the orientation of crossbar in the diagonal position of the balance and the orientation of the table top in the diagonal position.

Correct performance in the diagonal balance should be more strongly associated with correct performance at the diagonal table than with incorrect performance at the diagonal table.

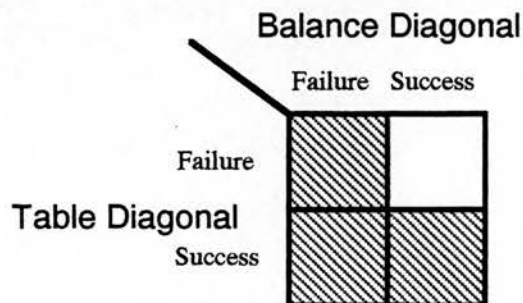


Figure 7.9 Model of collective décalage between responses to the diagonal balance and diagonal condition of the table according to hypothesis 2f (white cell is predicted to be empty).

7.4 METHOD

7.4.1 Design

There were three tasks: reproduction of the water-level in the round bottle, reproduction of the crossbar in the balance task, orientation of the table top. The order of presentation of the three tasks was randomized. Subjects could start with the round bottle then pass to the balance task and then pass to the table task, or begin with the balance then pass to the table and finish with the round bottle task, etc..

The design of the water-level conditions reproduces in part the design of Experiment 1. I decided to test children with only one container in order to reduce the number of drawings. The experiment involves three tasks and in order to keep the interview time to 15/20 minutes, it was thought best to eliminate one of the two water-level tasks. From previous results it has been established that the round container is usually solved correctly before the square one. Therefore we can consider that if subjects are to fail the round bottle they would certainly fail the square as well. Because of the young age of our sample, we also expect that subjects will already have difficulties in passing the diagonal round condition. Hence, it did not seem useful to test them on the square bottle as well.

For the round bottle condition, there are 8 possible angles of inclination. The angles are the following: 4 orthogonal 0° , 90° , 180° , 270° , and 4 diagonal 45° , 135° , 225° , 315° . Each subject will draw 4 lines representing the water-level and sees the bottle at 4 angles of tilt, 2 orthogonal and 2 diagonal. Therefore each subject sees each bottle in only half of the possible orientations. This is done in order to keep the number of drawings per child to 4, our subjects being even younger than those of previous experiments the number of drawings required had to be reduced. There are therefore 2 groups of subjects in each class, seeing a different set of orientations. Results from the previous study showed that there was no effect of group, we therefore assume that the two sets of angles are equivalent.

The balance task is an exact replication of Experiment 2. There are two conditions: an orthogonal condition when the balance is presented with the pivot upright perpendicular to the base, a diagonal condition when the pivot is tilted at 45 degrees with respect to the base. Children are always asked to do the orthogonal setting before the diagonal setting. Two settings for each condition were required. Therefore there were two settings with the balance upright and two settings with the pivot of the

balance tilted at 45 degrees. One setting was with the pivot at 45 degrees to the right and one setting with the pivot at 45 degrees to the left.

The table task is presented in two conditions the orthogonal condition when the base of the table is flat and horizontal and the diagonal condition when the base of the table is inserted in the wedge and is therefore tilted. All the subjects started with the flat base orthogonal condition, did one orientation and then passed to the diagonal condition where the table was inserted in the wedge. Here again the subjects were asked to orient the table top only once.

The following *independent variables* are manipulated:

- task complexity:
 - water-level task
 - balance task
 - table task
- angle of orientation of the objects supporting the horizontal element:
 - condition orthogonal
 - condition diagonal
- schooling/age group:
 - nursery (between 3 and 5 years)
 - primary 1 (between 5 and 6 years)
 - primary 2 (between 6 and 7 years)

The *dependent variable* is the number of correct orientations of the horizontal planes: table top, crossbar, water-level.

7.4.2 Statistical analysis

The two types of statistical treatments used in Experiment 2 have been used in this study, the first to analyse results between subjects and the second to verify the hypotheses of order of acquisition.

For the first level analysis:

- the McNemar test for significance of changes in related groups was employed to evaluate differences between the orthogonal and diagonal conditions for each of the three tasks;

- χ^2 test was employed to evaluate the differences between age groups' distribution of correct responses in each of the three tasks for the two conditions orthogonal and diagonal;

For the second level of analysis in order to test more directly the hypotheses of order, I adopted Hildebrand, Laing & Rosenthal Prediction Analysis of Cross-Classifications method that offers a means to evaluate which hypothesis of order of acquisition of two tasks better fits the data (an extended presentation is found in chapter 3). The 4 possible models are: synchronism, *décalage* in favour of Task1, *décalage* in favour of Task2, no systematic order between tasks.

For each hypothesis of order between two tasks, the predictions have been tested by computing a Del index which gives the improvement over chance after one or two cells supposed empty. The contingency tables include data based on allocating children in two categories: pass or fail.

7.4.3 Apparatus and Procedure

The experimental equipment consisted in 3 apparata for the 3 conditions of the task. The first element was the same as in the previous Experiment 1 and 2, and consisted of a round bottle sealed by a small cork with a capacity of half a litre. The round bottle was in fact, a glass sphere with a very small beaker.

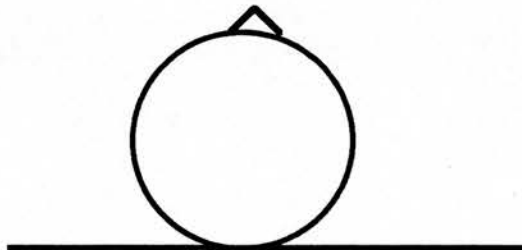


Figure 7.10: The round bottle

Children were asked to draw the level of the water on a card where was depicted the side outline of the bottle. The bottles were sitting on a line representing the table on which the real bottle was supported. On each card there was the bottle tilted to a certain angle, this corresponded to the angle to which the real bottle was oriented. The child was given a red pencil to draw with. The bottle remained visible and tilted at the corresponding angle while the child was drawing. The question was repeated for the 4

tilts of the bottle. Occasionally the experimenter asked the child to comment his drawing or to explain verbally or with a gesture the behaviour of the water.

Situation 2 employed the same apparatus as Experiment 2: a simplified version of a balance scale. The balance consisted of a wooden object composed of a base, a pivot, and a crossbar which could move on a pivot point and which had an indentation at each extremity in which a small object could be placed, these "holes" replaced the usual plates. When the two objects placed at each end of the crossbar were of equal weight the crossbar remained horizontal, on the contrary if the weights were different the crossbar swiveled on its center and tilted on the heaviest side.

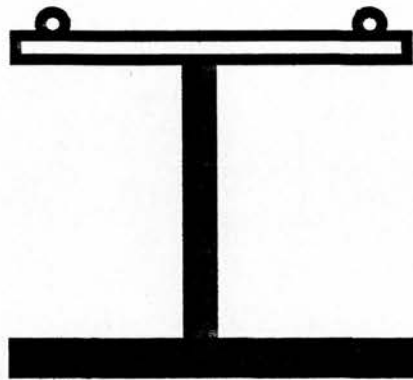


Figure 7.11 The balance in the upright /orthogonal position

A feature of this scale was that the pivot bar is itself mobile and could be inclined to the side at an angle of ± 45 degrees with respect to the base. The crossbar moved independently from the support hence, with equal weights in the holes the crossbar remained horizontal regardless of the inclination of the pivot.

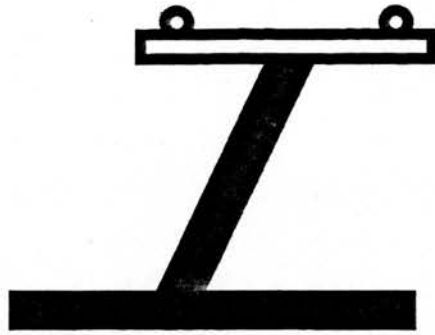
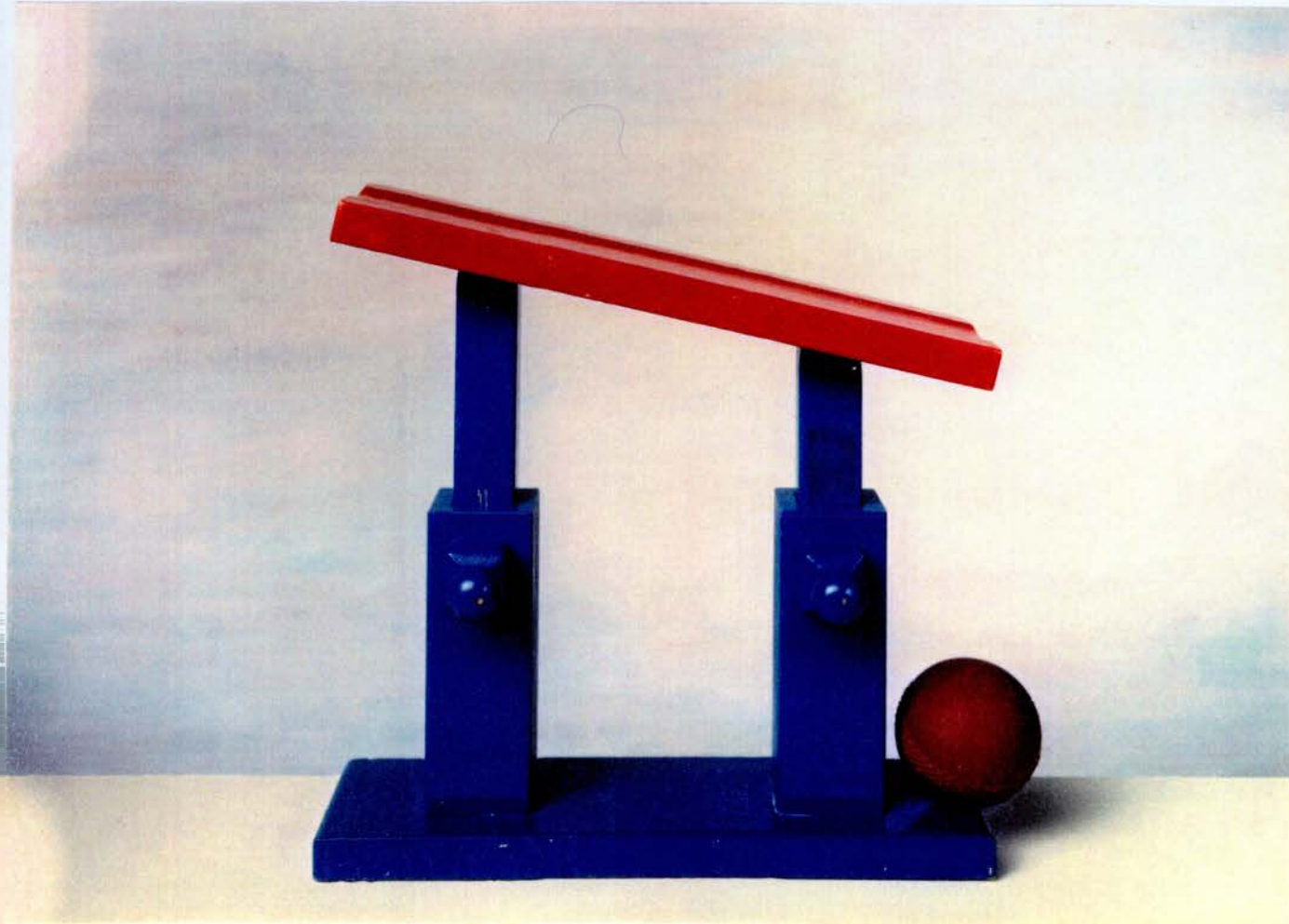
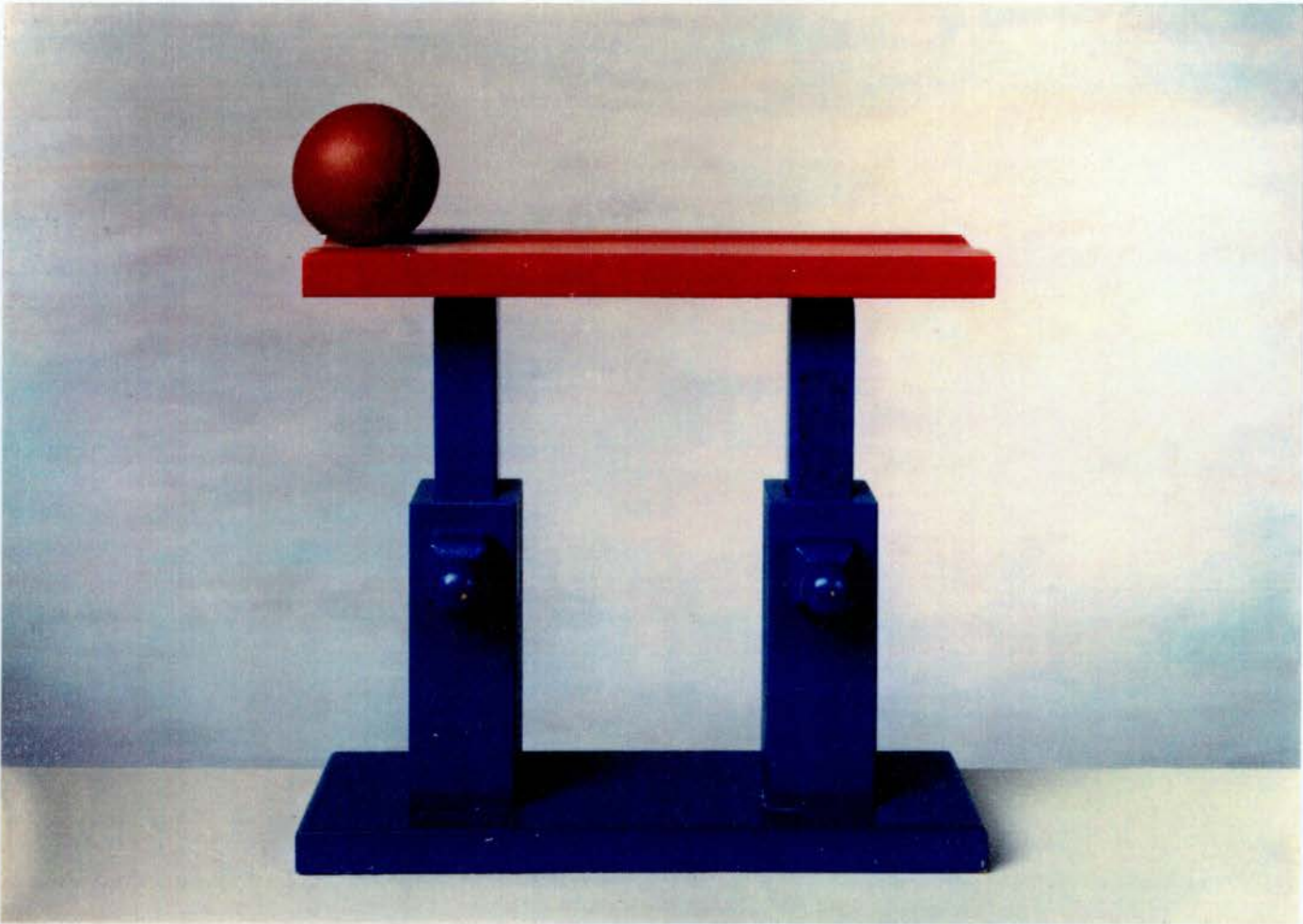


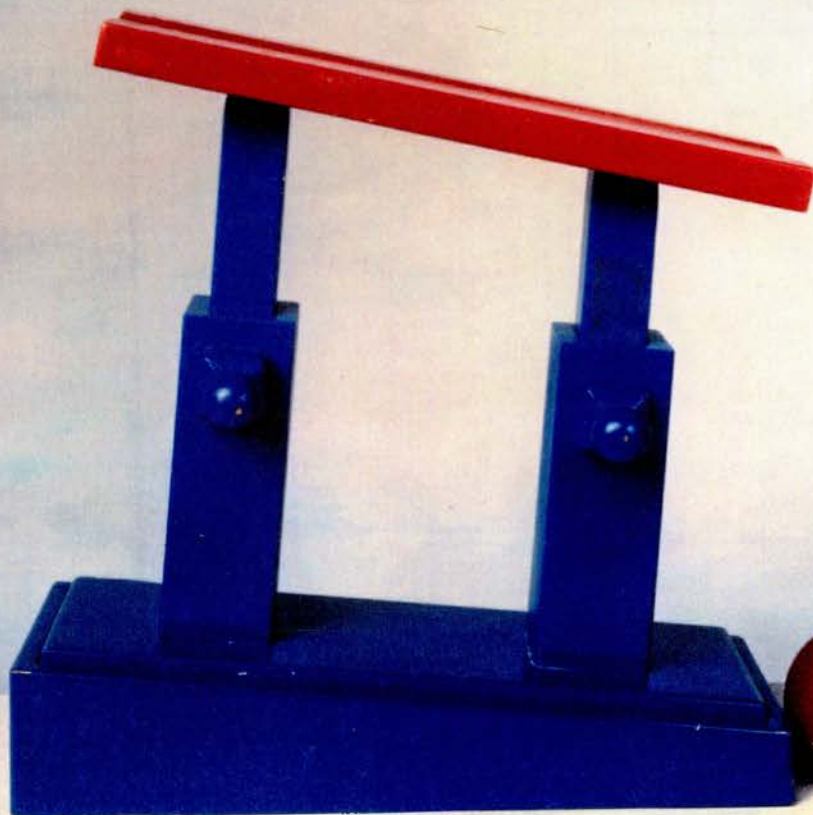
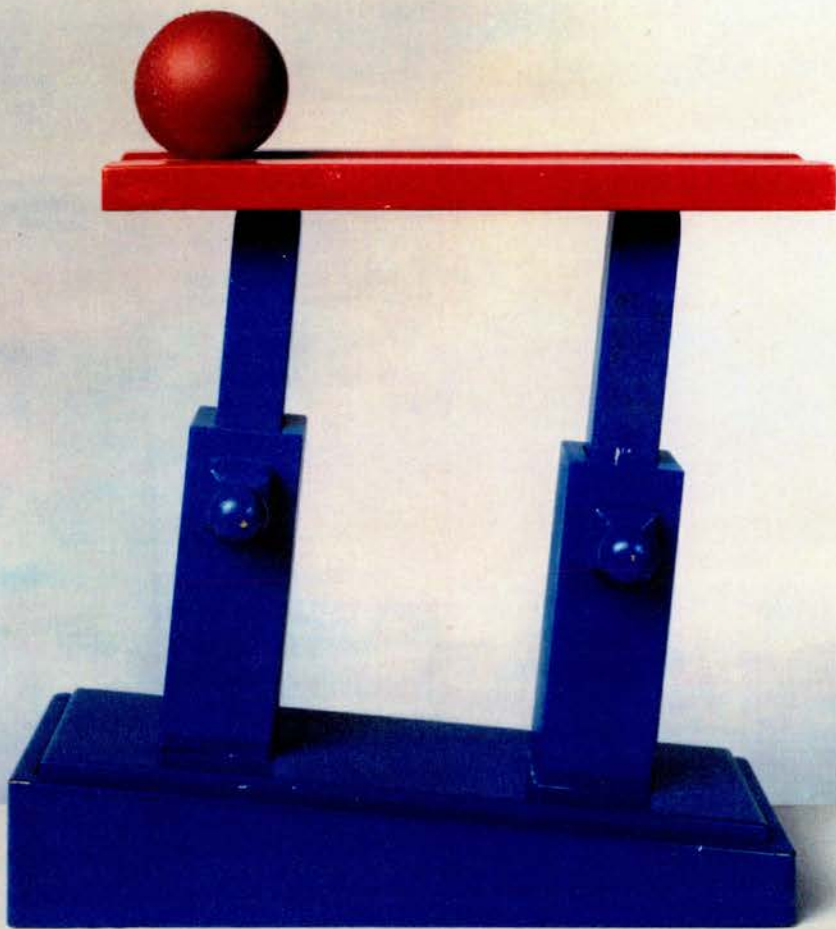
Figure 7.12 The balance in the tilted diagonal position.

Children were then presented with side drawings of the base and support and with a rectangle of red cardboard which represented the crossbar of the balance and were asked to place it on the drawing (it was sticky on one side and adhered to the cardboard of the card with the drawing of the base and pivot) in order to reproduce the position of the crossbar of the balance. The cards either showed the base with the perpendicular pivot, or the base with a tilted pivot, 45° to the right or to the left. The sticky rectangle could be put down then lifted and changed position.

Condition 3 involved a wooden object roughly the shape of a table. It was composed of a horizontal base from which departed two hollow columns that supported the legs of the table. Two square legs slid inside the hollow columns and could be fixed at different heights. On top of the legs was attached a rectangular plane. The base, legs and supports were blue the table top is red, the object measured 50 cm in height and 40cm in width (photographs next page). The base of the table could be inserted in an inclined base or wedge. The wooden wedge was also blue. When the table was put on the wedge, it was inclined at an angle of 45 degrees. In this case, in order to set the table top horizontally, one leg of the table had to be lengthened.

The subject was presented with the table and asked to play with it and try to move the legs up and down. The experimenter then blocked the legs at different heights in order to obtain an inclination in the table top. The experimenter held a little ball on top of the inclined plane and asked the child what would happen if she let the ball go. Children predicted that it would roll off, the experimenter tried and verified their prediction.





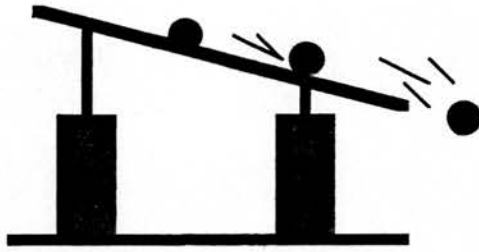


Figure 7.13 The table with the tilted top

Then the experimenter asked the child to fix the table top so that the ball would stay still and would not fall off. Children would slide the legs up and down until they were satisfied.¹³ The child was then given the little ball, he put it on the table and observed if it stayed still or rolled off.

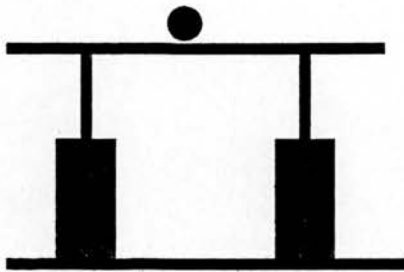
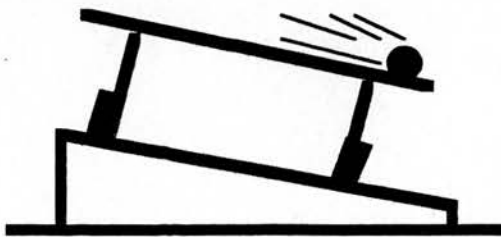


Figure 7.14 The table with the horizontal top

If the ball did not stay in equilibrium the child was asked to try again. The operation was repeated until the equilibrium was obtained or until the child gave up. The table was then put in the inclined base and the same question was asked again. This time the starting position was such that the legs were the same length, however the top plane was inclined because the base was inserted in the wedge.



¹³A small cylinder was inserted in one of the two hollow columns, in order to prevent children to simply push down the two legs until they reached the base of the columns. In fact the cylinder stopped the leg at a slightly higher level, children would therefore have to truly manipulate and align the two legs.

Figure 7.15 The table on the wedge with the tilted top

Again the child moved the legs of the table up and down until he was satisfied that the table top is in a position in which the little ball would be in equilibrium and would not fall off.

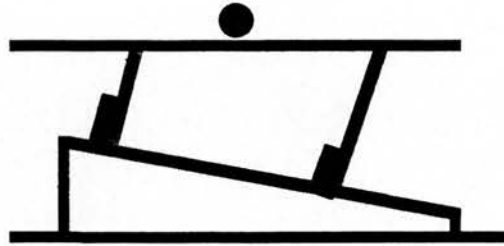


Figure 7.16 The table on the wedge with the horizontal top

7.4.4 Interviews

The interviews occurred in the classroom in a quiet corner cut off from the rest of the classroom by a high piece of furniture. The young age of the subjects made it preferable to conduct the interviews in the classroom rather than take them to a separate room. Children were seen individually for a session of 15/20 minutes. The experimenter sat in front of the child and moved the various pieces of equipment and asked the questions. The experimenter also took notes of the subject's remarks and any exceptional behaviour.

7.4.5 Subjects

Subjects were 60 children from a school in central Edinburgh. Age ranged from 3 years to 7 years, classes Nursery, Primary 1 and Primary 2. There were 20 subjects from each class. Mean age for Nursery children was 4,3 years ($SD = .32$); mean age for Primary 1 was 6,2 years ($SD = .24$); and mean age for Primary 2 was 7,1 years ($SD = .22$).

7.4.6 Measure

For condition 1, drawing the water level in the round container, responses were measured as angles off the horizontal. Measures of the subject's drawings were taken with a protractor. The orientations of the lines produced could span from 0 to 180 degrees. When lines were not drawn and children produced scribbles or coloured the totality of the figure, the drawings were coded as errors of 180 degrees. Wavy lines were approximated to the best fit, and this average line was measured with the protractor.

Any line tilted 10 degrees or less was considered correct. The 10 degrees allowing for drawing errors. Over 10 degrees off 0 the water line was considered incorrectly oriented. In the contingency tables, where subjects were classified as successful or unsuccessful, the criterion for failure was that a subject had drawn one or more incorrect water-levels.

For Task 2 the balance task, responses were measured as angles off the horizontal. The angle formed by the pivot and the rectangular "sticker" was taken with a protractor. The particular experimental situation did not allow measures as precise as the ones in the water-level task. The outlines of the balance on which the "stickers" were set and the stickers themselves were used repeatedly for all the subjects. In other words it was not possible as with the water-level drawings, to preserve each child's response for further scrutiny. The measures of the 4 settings of the sticker on the 4 outlines had to be carried out at the end of each interview. For this reason the 4 settings were measured solely to control whether the angle was over 10 degrees and then categorised as correct or wrong. Any setting tilted 10 degrees or less was considered correct. Over 10 degrees off the horizontal the orientation was considered incorrect.

In the contingency tables, where subjects were classified as successful or failing in each of the two conditions, the criterion for failure was that a subject had done one or more incorrect setting for each condition.

For condition 3, orienting the table top horizontally, responses were measured as successful or failed according to the fact that a subject had been capable of setting the table top in such a way to avoid the little ball to roll off. The stability of the ball was considered an exact measure of the orientation of the table top. Subjects who failed the task were those who could not set the table top correctly even after a few trials, 5 trials were the limit. Were scored correct those subject that succeeded in placing the table

top horizontally even after a few trials. No discrimination was made between subjects succeeding the task after 1 trial and subjects succeeding after 3 trials, for example.

7.5 Results

7.5.1 First level analysis: comparison between the orthogonal and the diagonal conditions

The first analysis of the results was conducted to verify our hypotheses on the difference between the two conditions diagonal/orthogonal within each task.

		Diagonal	
		Failure	Success
Orthogonal	Success	11	24
	Failure	25	0
		Bottle	

		Diagonal	
		Failure	Success
Orthogonal	Success	12	46
	Failure	2	0
		Table	

		Diagonal	
		Failure	Success
Orthogonal	Success	45	12
	Failure	3	0
		Balance	

Table 7.1 Correct and failed orientations for the orthogonal and diagonal conditions of the 3 tasks: bottle, table and balance.

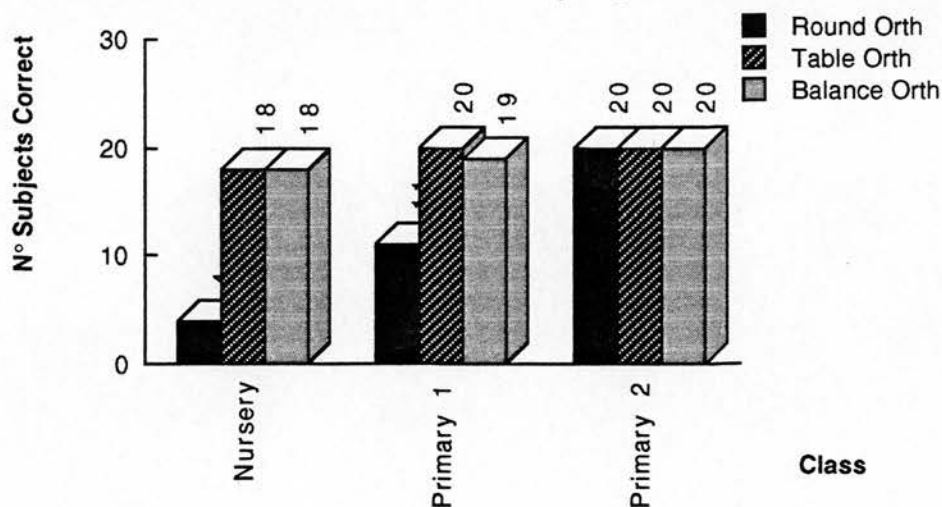
Inspection of these tables indicates that the frequencies of correct and incorrect subjects varied considerably between the orthogonal and diagonal conditions for each task. In each case it appears that the orthogonal condition elicited significantly more correct responses than the diagonal condition. Specifically, the McNemar test for significance of changes in related groups indicated that in every case there was a significant difference between condition orthogonal and condition diagonal.

In the bottle task, the difference in responses between orthogonal and diagonal was significant ($\chi^2 = 8.1$ df.=1 $p < .01$). In the table orientation task, the difference between the orthogonal and diagonal condition was also significant ($\chi^2 = 10.08$ df.=1 $p < .01$). In the balance task, there were significantly more correct responses in the orthogonal than the diagonal condition ($\chi^2 = 43$ df.=1 $p < .001$).

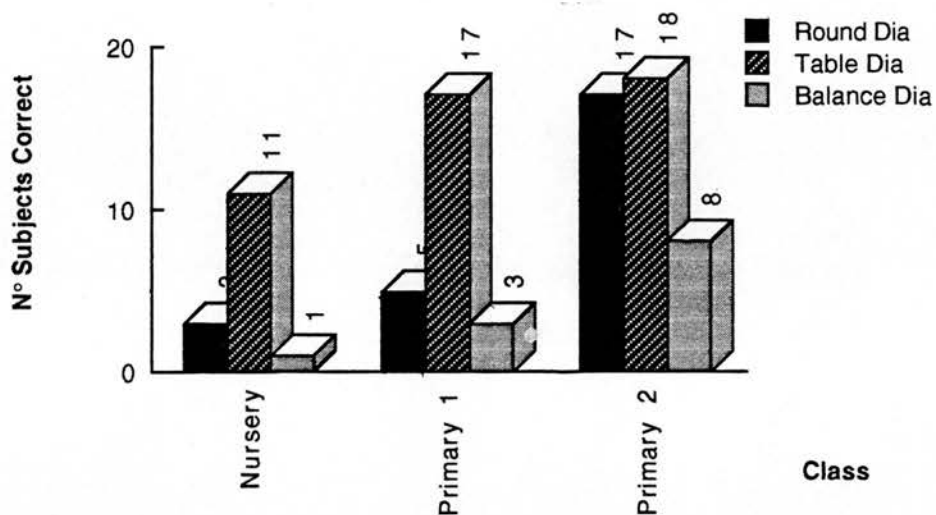
In conclusion we can reject the null hypothesis that there is no change in subjects' responses to the two conditions for all three tasks. There is a significant change in the number of correct responses between the orthogonal and diagonal conditions. It also appears from the tables that the orthogonal condition in every case elicits a greater number of correct responses.

7.5.2 Age group comparison

A second analysis was carried out to determine the age period in which some developmental change occurs on the basis of the difference in age groups' performance.



Graph 7.1 Number of subjects in each class performing correctly at each of the three tasks in the orthogonal condition.



Graph 7.2 Number of subjects in each class performing correctly at each of the three tasks in the diagonal condition.

The following tables present the contingency tables of the children's correct and wrong responses to each task condition as a function of age. A χ^2 test is computed on each contingency table. Occasionally a further breakdown of the table is required to identify the precise age groups in which the performance varies significantly.

Bottle Orthogonal				
age Res	Nurs.	P1	P2	
Succ.	4	11	20	
Fail	16	9	0	

Bottle Diagonal				
age Res	Nurs.	P1	P2	
Succ.	3	5	17	
Fail	17	15	3	

Table 7.2 Correct and failed orientations of the water-level by age group in the two conditions orthogonal and diagonal.

Condition orthogonal: $\chi^2 (2, N=60) = 26.65$ $p < 0.001$ There is a significant difference from chance distribution in the distribution of children's responses in the three age groups. This indicates a significant increase in correct responses in the three

age groups for the orthogonal condition of the water-level task. The round bottle orientation task, showed a significant improvement with age in the number of correct responses, for both conditions diagonal and orthogonal. Most nursery children and Primary 1 tend to scribble rather than represent the water level with a line and for this reason the improvement is visible also in the case of the orthogonal condition where subjects pass from scribbling to drawing the water-level as a horizontal line.

Condition diagonal: $\chi^2 (2, N=60) = 23.64$ $p < 0.001$ There is a significant difference from chance distribution in the distribution of children's responses in the three age groups. This indicates a significant increase in correct responses in the three age groups for the diagonal condition of the water-level task.

In the balance task no difference between the 3 age groups was found in the orthogonal condition as the great majority of subjects in each class were successful in orienting the scale bar. 100% of Primary 2, 95% of primary 1, and 90% of Nursery children succeeded in the orthogonal condition.

Differences are found however in the solution of the diagonal condition.

Balance Orthogonal				Balance Diagonal			
age Res	Nurs.	P1	P2	age Res	Nurs.	P1	P2
Succ.	18	19	20	Succ.	1	3	8
Fail	2	1	0	Fail	19	17	12

Table 7.3 Correct and failed orientations of the balance by age group in the two conditions orthogonal and diagonal.

Orthogonal condition: $\chi^2 (2, N=60) = 0.1$ $p > .95$ The χ^2 test has proved not significant. Diagonal condition: $\chi^2 (2, N=60) = 8.12$ $p < 0.02$ There is a significant difference from the chance distribution in the distribution of children's responses in the three age groups. A significant increase in correct responses with age is found for the diagonal condition of the balance task.

For Task 3, the table, in the orthogonal orientation practically all the subjects of all classes succeeded in the orthogonal condition. No age difference was observed for this condition. 100% of Primary 1 and Primary 2 succeeded the task and 90% of

Nursery children succeeded in orienting the table top in the orthogonal condition. Differences are found however in the solution of the diagonal condition.

Table Orthogonal				Table Diagonal			
age Res	Nurs.	P1	P2	age Res	Nurs.	P1	P2
Succ.	18	20	20	Succ.	11	17	18
Fail	2	0	0	Fail	9	3	2

Table 7.4 Correct and failed orientations of the table by age group in the two conditions orthogonal and diagonal.

For the orthogonal condition because of the ceiling effect, with zero scores in 2 cells, it was not possible to compute a valid χ^2 . The binomial test on the results of Nursery and Primary 1 resulted non significant and allows us to conclude that there is no significant improvement with age in the number of correct responses in the orthogonal condition of the task..

Diagonal condition: $\chi^2 (2, N=60) = 8.01$ $p < 0.02$ There is a significant difference from the chance in the distribution of children's responses in the three age groups. There is therefore a significant increase in correct responses with age for the diagonal condition of the balance task.

In conclusion, significant changes with age are observed in all three tasks for the diagonal conditions, and in the orthogonal condition of the bottle where children pass from scribbling to representing the water-level with a line which is immediately oriented correctly.

7.5.3 Prediction analysis of acquisition orderings

The testing of the specific developmental hypotheses of concurrency and *décalage* in the solution of the 2 conditions, orthogonal and diagonal, of the water-level task, the 2 conditions of the balance task, and the 2 conditions of the table task, was carried out by a statistical analysis of children's performance across pair of tasks using Hildebrand, Laing & Rosenthal's Prediction Analysis of Cross Classifications.

Hypothesis 1a: collective décalage between orthogonal and diagonal orientations for the round bottle.

The hypothesis was made that there exists a collective décalage between orthogonal and diagonal orientations of the round bottle, in favour of the orthogonal. The triangular hypothesis associated with collective décalage in favour of orthogonal solutions over diagonal solutions predicts that the cell corresponding to failure in the orthogonal and success at the diagonal be empty. In the language of prediction analysis we predict that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models.

The table below indicates the number of subjects having failed or succeeded in the orthogonal and diagonal positions for the bottle.

		Bottle Diagonal	
		Failure	Success
Bottle Orthogonal.	Failure	25	0
	Success	11	24

Table 7.5: Contingency table of success and failure in the orientation of the water-level for orthogonal and diagonal orientations of the bottle

Mod 3: Del = 1 ($z = e$, $p < 0.00001$)

$z_{M1-M3} = -3.89$ $p = 0.00007$

$z_{M2-M3} = -5.27$ $p = 0.00003$

Model 3, décalage in favour of the orthogonal bottle has proved significant. The Del is equal to 1 and the model is a significantly better predictor than any of the other two models, concurrence or opposite collective décalage. This result indicates that when children solve correctly the orthogonal condition they can either fail or succeed in the diagonal condition. However if they fail the orthogonal they systematically fail the diagonal as well. The results confirm once again the anticipation of the orthogonal condition in the water-level task that has been found in the two preceding experiments.

Hypothesis 1b: collective décalage between orthogonal and diagonal orientations of the balance.

The hypothesis of a collective décalage in the solution of the orthogonal and diagonal orientations of balance, in favour of the orthogonal corresponds to the prediction that the cell corresponding to failure in the balance diagonal and success at the orthogonal balance be empty, and thus that Model 3, décalage in favour of Task1 will be significant and a significantly better predictor than the other 2 models. The table below indicates the number of subjects having failed or succeeded the orthogonal and diagonal positions for the balance.

		Balance	
		Diagonal	
Balance	Failure	3	0
	Success	45	12

Table 7.6 Contingency table of success and failure in the reproduction of the orientation of the crossbar in the conditions balance orthogonal and balance diagonal.

Mod 3: Del =1 $z=e$, $p<0.00001$

$z_{M1-M3} = -58.7$ $p<0.00001$

$z_{M2-M3} = -115.9$ $p<0.00001$

Model 3, décalage in favour of Task1, i.e.orthogonal balance has proved significant. The Del is equal to 1 and the model is a significantly better predictor than any of the other two models, concurrence or opposite collective décalage. This result indicates that when children solve correctly the orthogonal condition they can either fail or succeed in the diagonal condition. When they fail the orthogonal they systematically fail the diagonal as well. This result confirms the evidence found in Experiment 2 on the anticipation of the orthogonal condition on the diagonal condition of the balance.

Hypothesis 1c: collective décalage between orthogonal and diagonal orientations of the table.

The hypothesis that there exists a collective décalage in the solution of the orthogonal and diagonal orientations of the table, in favour of the orthogonal, predicts that the cell corresponding to failure in the table diagonal and success at the orthogonal table be empty. In the language of prediction analysis we predict that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models. The table below indicates the number of subjects having failed or succeeded the orthogonal and diagonal positions for the table.

		Table Diagonal	
		Failure	Success
Table Orthogonal	Failure	2	0
	Success	12	46

Table 7.7 Contingency table of success and failure in the reproduction of the orientation of the table in the conditions table orthogonal and table diagonal.

Mod 3: Del = 1 $z=e$ $p<0.00001$

$z_{M1-M3} = -6.3$ $p<0.00003$

$z_{M2-M3} = -11.4$ $p<0.00003$

Model 3, décalage in favour of Task1 i.e. orthogonal table has proved significant. The Del is equal to 1 and the model is a significantly better predictor than any of the other two models: concurrence or the opposite collective décalage. The result indicates that when children solve correctly the orthogonal condition they can either fail or succeed in the diagonal condition, however if they fail the orthogonal they systematically fail the diagonal as well.

Hypothesis 2a: concurrence between bottle orthogonal and orthogonal balance.

The hypothesis that the ability to reproduce the orientation of the crossbar when the balance is orthogonal concurs with the ability to reproduce the orientation of the water-

level when the round container is orthogonal predicts that Model 1 concurrence between Task1 and Task 2 will be significant and a significantly better predictor than the other 2 models. Children are expected to either fail both tasks or succeed in both. The table below indicates the number of subjects having failed or succeeded the orthogonal and positions for the balance and for the bottle.

		Bottle Orthogonal	
		Failure	Success
Balance Orthogonal	Failure	3	0
	Success	22	35

Table 7.8 Contingency table of success and failure in the reproduction of the orientation of the balance orthogonal and the bottle orthogonal.

The concurrence prediction has not been confirmed. Model 1 of concurrence has not proved significant and has therefore falsified our hypothesis. As a consequence a further hypothesis has been made of collective décalage in favour of balance orthogonal. Model 3 has therefore been tested.

Mod 3: Del = 1 $z = e$ $p < 0.00001$

$z_{M1-M3} = -11.5$ $p < 0.00003$

$z_{M2-M3} = -21.5$ $p < 0.00003$

Model 3, décalage in favour of the balance orthogonal task has proved significant. The Del is equal to 1 and the model is a significantly better predictor than any of the other two models: concurrence or the opposite collective décalage. The result indicates that when children solve correctly the orthogonal condition for the balance they can either fail or succeed in the orthogonal condition of the bottle, however if they fail the orthogonal balance they systematically fail the orthogonal bottle as well.

Hypothesis 2b : concurrence between bottle orthogonal and orthogonal balance.

The hypothesis was made that the ability to reproduce the orientation of the crossbar when the balance is diagonal is concurrent with the ability to reproduce the orientation

of the water-level when the bottle is diagonal. Model 1, concurrence between Task 1 and Task 2, is predicted to be significant and a significantly better predictor than the other to models. Children are expected to either fail both tasks or succeed in both. The table below indicates the number of subjects having failed or succeeded the diagonal positions for the balance and for the bottle.

		Bottle Diagonal	
		Failure	Success
Balance Diagonal	Failure	32	16
	Success	4	8

Table 7.9 Contingency table of success and failure in the reproduction of the crossbar in the condition balance diagonal and of the water-level in the condition bottle diagonal.

Mod1: Del= .24 $z=2$ $p=0.022$
 Mod2: Del= .44 $z=2.1$ $p=0.017$
 Mod 3: Del = .16 $z=1.8$ $p=0.035$
 $z_{M1-M2} = -1.6$ $p=0.05$
 $z_{M1-M3} = 2.14$ $p < 0.017$
 $z_{M2-M3} = 1.7$ $p < 0.044$

The three models are significant and none of them seems to be a better predictor than the other. We conclude that there is a weak concurrence between the two conditions. Note that this result mirrors the result of the comparison of these two conditions in Experiment 2. These results are not conclusive although they show a tendency for the two tasks to be solved concurrently in the diagonal condition.

Hypothesis 2c : collective décalage between bottle orthogonal and table orthogonal.

The hypothesis that there exists a collective décalage in the solution of the orthogonal orientations of the table and bottle, in favour of the table, predicts that the cell

corresponding to failure in the table and success at the bottle be empty and thus that that Model 3, décalage in favour of Task1 will be significant and a significantly better predictor than the other 2 models. The table below indicates the number of subjects having failed or succeeded the orthogonal positions for the table and bottle.

		Bottle Orthogonal	
		Failure	Success
Table Orthogonal	Failure	2	0
	Success	23	35

Table 7.10 Contingency table of success and failure in the reproduction of the water-level in the condition bottle orthogonal and of the orientation of the table top in the condition table orthogonal.

Mod 3: Del = 1 $z=e$ $p<0.00001$

$z_{M1-M3} = -14.4$ $p < 0.00003$

$z_{M2-M3} = -27.5$ $p < 0.00003$

Model 3, décalage in favour of Task1 i.e. orthogonal table has proved significant. The Del is equal to 1 and the model is a significantly better predictor than than any of the other two models, concurrence or opposite collective décalage. The result indicates that when children solve correctly the orthogonal condition for the table they can either fail or succeed in the orthogonal condition of the bottle. However when they fail the orthogonal table they systematically fail the orthogonal bottle as well.

Hypothesis 2d: collective décalage between bottle diagonal and table diagonal.

The hypothesis that there is a collective décalage in the solution of the diagonal orientations of the table and the bottle tasks, in favour of the table predicts that the cell corresponding to failure in the table diagonal and success at the diagonal bottle be

empty. In the language of prediction analysis we predict that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models. The table below indicates the number of subjects having failed or succeeded the diagonal positions for the table and bottle.

		Bottle Diagonal	
		Failure	Success
Table Diagonal	Failure	11	3
	Success	25	21

Table 7.11 Contingency table of success and failure in the orientation the table top in the condition table diagonal and the reproduction of the water-level in the condition bottle diagonal

Mod 3: Del = .93 z=e p<0.00001

zM1-M3= - 1.69 p < 0.054

zM2-M3= -1.76 p< 0.044

Model 3, décalage in favour of Task1 i.e. diagonal table has proved significant. The Del is significant and the model is a significantly better predictor than than any of the other two models, concurrence or opposite collective décalage. The result indicates that when children solve correctly the diagonal condition for the table they can either fail or succeed in the diagonal condition of the bottle, however if they fail the diagonal table they systematically fail the diagonal bottle as well.

Hypothesis 2e: collective décalage between table orthogonal and balance orthogonal.

The hypothesis that there is a collective décalage in the solution of the orthogonal orientations of the table and balance, in favour of the table predicts that the cell corresponding to failure in the table orthogonal and success at the orthogonal balance be empty and that Model 3, décalage in favour of Task1 will be significant and significantly better than the other 2 models. The table below indicates the number of subjects having failed or succeeded the orthogonal positions for the table and balance.

		Balance Orthogonal	
		Failure	Success
Table Orthogonal	Failure	2	0
	Success	1	57

Table 7.12 Contingency table of success and failure in the reproduction of the crossbar in the condition balance orthogonal and of the table top in the condition table orthogonal.

Mod 1: Del = .79 $z = 3.8$ $p = 0.00007$

Mod 2: Del = .65 $z = 2.9$ $p = 0.0018$

Mod 3: Del = 1 $z = e$ $p < 0.00001$

The three models have proven significant. It is not possible to decide which of the three is a better predictor as the comparison between models does not provide any significant difference. As in previous similar situations we conclude that there is a weak concurrence between the two tasks. Since 95% of the subjects succeed in both tasks, nothing conclusive can be said as to the order in which the two tasks are acquired. Nevertheless, within this sample a clear stage emerges in which the two tasks are solved together.

Hypothesis 2f: collective décalage between table diagonal and balance diagonal.

The hypothesis that there is a collective décalage in the solution of the diagonal orientations of the table and balance, in favour of the table predicts that the cell corresponding to failure in the table diagonal and success at the diagonal balance be empty and that Model 3, décalage in favour of Task1, table diagonal, will be significant and significantly better than the other 2 models. The table below indicates the number of subjects having failed or succeeded the diagonal positions for the table and balance.

		Balance Diagonal	
		Failure	Success
Table Diagonal	Failure	14	0
	Success	34	12

Table 7.13 Contingency table of success and failure in the reproduction of the crossbar in the condition balance diagonal and of the table top in the condition table diagonal.

Mod 3: Del = 1 $z=e$ $p<0.00001$

$z_{M1-M3} = -17.2$ $p<0.00003$

$z_{M2-M3} = -31.8$ $p<0.00003$

Model 3, décalage in favour of Task1 i.e. diagonal table has proved significant. The Del is equal to 1 and the model is a significantly better predictor than any of the other two models, concurrence or opposite collective décalage. The result indicates that when children solve correctly the diagonal condition for the table they can either fail or succeed in the diagonal condition of the balance. When they fail the diagonal balance they systematically fail the diagonal table as well.

I wish to present the responses to the orthogonal bottle and diagonal table conditions although I did not make any preliminary hypothesis on this acquisition order.

However, given the décalage that has appeared between the orthogonal condition of the bottle and the other orthogonal tasks, it seems important to assess whether success in the diagonal condition of the table precedes or follows success in drawing the water-level correctly in the orthogonal bottle.

		Table Diagonal	
		Failure	Success
Bottle Orthogonal.	Failure	9	16
	Success	5	30

Table 7.14 Contingency table of correct and failed orientations of the water-level in the condition bottle orthogonal and of the table top in the condition table diagonal.

Mod 1: Del = .231 $z=1.92$ $p< 0.0274$

Mod 2: Del = .165 $z=1.79$ $p< 0.0395$

Mod 3: Del = .387 $z=2.5$ $p< 0.0287$

$z_{M1-M3} = -1.44$ $p< 0.065$

$z_{M2-M3} = -1.60$ $p<0.0523$

$z_{M1-M2} = 1.93$ $p<0.0278$

The three models have proved significant and there is no Model that is a significantly better predictor than the others although Model 3, *décalage* in favour of Table Diagonal, is a better predictor than Model 2, *décalage* in favour of Bottle Orthogonal. We can conclude that there is a weak concurrence between the two conditions and a tendency for the Table Diagonal to be solved before the Orthogonal Bottle. This result is interesting because it shows that there is at least one diagonal condition of our tasks that seems to precede or concur with the resolution of one of the orthogonal conditions and that there is no anticipation of the orthogonal condition on the diagonal table condition. The consequences of this result will be discussed in the next sections. We can anticipate however that this result challenges the view that there is one stage in which subjects are "object centered" which is followed by a subsequent "framework centered" stage. The fact that the orthogonal bottle does not precede the diagonal table seems in fact to exclude the possibility of envisaging a unique stage in which all the orthogonal conditions are solved before any of the diagonal conditions.

7.6 Discussion of the results

In this experiment we set out with a number of questions and predictions. The discussion of the results will follow the order in which the different hypotheses were formulated in the first part of the chapter. I will first consider the patterns of acquisition that have emerged in the water-level and balance tasks, and therefore address the question of how this younger group of subjects has performed in the two tasks and the order in which they are acquired. Then I will discuss the table task and enrich the description of these results with a few qualitative observations. Finally I will consider the patterns of responses and the orders of acquisition that have emerged across the various tasks. Before moving to the analysis of the response patterns within the three tasks, I would like to briefly highlight a few general points that have emerged from this study.

The "orthogonal effect" has been found once again in the new table task and confirms the findings of previous experiments with the bottles and balance. In fact, the orthogonal condition of the table elicits a significant greater number of correct responses than the diagonal condition. Therefore, the difference between the performance in the orthogonal and diagonal conditions has been confirmed in the three tasks. We thus have the grounds to consider this effect a general developmental phenomenon and a robust finding.

However it has appeared that not all the orthogonal conditions are correctly solved at the same time. The results at the water-level task in the orthogonal bottle condition exhibited an improvement with age, while in the other two tasks the orthogonal conditions were solved correctly by 90% of the subjects starting from the Nursery level. As we will see this improvement in children's responses to the orthogonal bottle corresponds to the transition from drawing the water-level as a scribble to representing it with a line.

Furthermore, the age group analysis showed that there was a significant increase in correct responses with age for the three tasks in the diagonal conditions. However, the increase is different for the three tasks; in the table diagonal there is a shift from 50% to 100% of correct responses from nursery to primary 2, while in the balance task the shift is from 5% of correct responses in nursery to only 40% of correct answers in primary 2. In the bottle task, the improvement is of the order of 70% and goes from 10% of correct answers in the Nursery class to 80% in Primary 2. This goes to show that the transition from success only in the orthogonal condition to success also in the diagonal condition does not happen once and for all the tasks simultaneously. On the

contrary the shift from orthogonal to diagonal happens first for the table task and later for the balance and bottle tasks. This is an important fact to bear in mind as it dismisses from now the hypothesis that there are two subsequent and independent levels of spatial organisation; the first in which children can only orient a plane horizontally in the orthogonal conditions (an object centered stage) and the second in which a plane is correctly oriented in the diagonal conditions (a framework centered stage). From now we can suggest that the stage sequence is more complex and does not reflect such a straightforward transformation.

7.6.1 The water-level task

As expected most of the younger subjects failed the water-level task. In fact 75% of Nursery children and 50% of Primary 1 children failed both the orthogonal and the diagonal conditions of the bottle task. Their failure was essentially due to the fact that they represented the water-level as a scribble rather than a line; none of the 25 subjects who fail the orthogonal condition represent the water-level with a line. In other words, none of their errors consist in the incorrect orientation of the line, as would be the case if for example they were doing inversion errors in the 90 degrees orientations of the bottle. When subjects start representing the water-level with a line they immediately orient it horizontally for the orthogonal positions of the container. This result is extremely important to confirm the characterisation of children's spatial representations at this stage of development, we proposed in previous chapters. When children start drawing a line they can orient it correctly even though it is only when the internal frame of reference coincides with the external frame of reference.¹⁴

The "orthogonal effect" was confirmed once again, as a significant number of subjects failed the diagonal condition while succeeding the orthogonal condition. This result has been sufficiently discussed in previous sections not to require further comment.

7.6.2 The balance task

Results at the balance task are a perfect replication of the results found in Experiment 2, the majority of subjects regardless their age were successful in the orthogonal condition and the anticipation of the orthogonal on the diagonal condition was

¹⁴ This result may suggest that from the beginning linear representations embed and express a direction or orientation.

confirmed. The novelty of these results is the age of the children which can correctly solve the orthogonal condition. It has emerged that 90% of the nursery children could orient the crossbar horizontally. This very precocious (and unexpected) competence needs an explanation.

In experiment 2, I introduced a pretest in which children were tested as to their understanding of the working of the balance. Children that could not correctly predict the inclination of the crossbar when equal and different weights were put on the scales, were excluded from the experiment. In this experiment I did not exclude any subject after the pretest, not because all the subjects understood the task but on the contrary because none of the Nursery and Primary 1 subjects understood it. I decided to test them despite their failure to understand the full working of the balance in order to see what they would do. I simply asked them to put the sticker on the card so it would look like the balance in front of them. As the balance was visible in front of them children were very good at copying the orientation of the crossbar on the space of the card. Primary 2 subjects on the contrary all understood the principle of the balance and could predict correctly how the crossbar would position itself when equal or different weights are put in the "holes". In sum, although there is a difference in the older and younger children's understanding of the physical properties of the balance, the output of behaviour is the same as the young subjects can reproduce correctly the configuration of the balance in its upright position.

Where the difference in understanding of the physical knowledge is felt more clearly in the responses to the diagonal condition. In fact 75% of the subjects of this study, fail the diagonal condition while succeeding the orthogonal. Even in Primary 2, 60% of the children fail this condition. Contrary to Experiment 2, where the errors of the older subjects consisted mainly in settings of 20 to 30 degrees off the horizontal, most of the incorrect settings of these subjects consist in perpendicular errors. In fact subjects tend to place the sticky crossbar perpendicular to the pivot even when the pivot is tilted at 45 degrees. This result can be put in relation with the evidence reported by Piaget and by some replication studies of children's inversion errors in the water-level task. In those studies it was observed that young children were drawing the water-level perpendicular to the sides or always parallel to the bottom even when the bottle was on its side. These results were reported for the situations in which the child was asked to draw the water in anticipation when the bottle was covered. In fact in my experiments with the bottle visible I found very few examples of this type of error.

These "inversion" errors were explained in the literature as a persistence of the canonical relation between water and bottle as if the child treated the task as one of

rotation in which the liquid is not mobile. The fact that Nursery children which do not understand how the balance works and that the crossbar is dynamically independent, do the same type of errors can shed some light on these types of errors for both tasks. When the child does not realise the dynamic independence of the plane, be it the water-level or the crossbar, he represents it in the same relation it initially holds with its supporting elements. The plane is therefore represented perpendicular to the sides or to the pivot as if a rotation of the object as whole did not affect the relation of the plane with the object; as if the plane were not dynamically independent.

On the contrary children who represent the plane at an incorrect orientation but at an angle greater or smaller than 90 degrees with its support, seem to realise the independence of the crossbar or the liquid but are unable to determine the correct orientation. In fact, subjects of Experiment 2 and some of the Primary 2 children in this experiment that understood the working of the balance, produced settings of this second sort. Similarly children who can see the water realise that the liquid moves and therefore do not maintain a fix relation between the bottle and the liquid.

7.6.3 The order between the balance and the water-level tasks

In the previous experiment we were not in the position to establish the order in which were acquired the orthogonal conditions of the water-level and the balance tasks, as all the subjects were able to solve both of them correctly. In this experiment however, a clear result has emerged. The orthogonal balance precedes the orthogonal bottle. In fact as young children cannot represent the water-level with a line they fail the orthogonal condition of the water-level task as well as the diagonal, on the contrary we have observed that they are in majority successful in reproducing the orientation of the crossbar. This result can be given various interpretations. We could suppose that the physical context of the task is playing a role, however as I discussed above Nursery and Primary 1 children have a limited understanding of the dynamic properties of the balance and approach the task essentially as a problem of reproducing the configuration of the balance. Remark that these children are probably giving the same interpretation of the water-level problem. Therefore the most plausible explanation of the anticipation of the balance orthogonal on the bottle orthogonal is that at this early stage when the child is given the element to orient i.e. the sticker, he can orient it correctly while he cannot yet draw a line and give it a direction and orientation. Ibbotson & Bryant (1976, presented in Chapter 4) report having successfully used a similar response means (putting down on the page a prepared little bar) in their investigation of

3 to 5 years old subjects' competence in copying oriented lines. This method was employed to avoid the difficulties in scoring children's drawings of lines which were found to be not straight enough. Therefore by asking children to lay down a little straight piece of wire on the page, they measured young children's ability to depict the angle formed by the wire and the baseline drawn on the page.

With the very young age group of our study therefore, the response means of the balance task has been found to be more adequate than drawing. In the situation in which the outline representing the plane is given, our younger subjects were capable of orienting and reproducing the horizontality of the crossbar when the balance was in the orthogonal position.

As far as the diagonal orientations of the two tasks are concerned, the weak concurrence found in Experiment 2 was confirmed with maybe a slight tendency to be more correct in the bottle task than in the balance task. It is surprising to observe that the bottle "catches up" with the balance. Although children can represent the horizontal crossbar earlier than the water-level in the orthogonal condition, there is a long delay before they acquire the competence to represent the correct orientation of the crossbar in the diagonal condition. It would appear, that the transition to a level in which the diagonal condition is solved as well, corresponds to a more general shift in spatial representations (of which is evidence the concurrent solution to the two tasks), which is then independent from the means of response.

7.6.4 The table task

The table task has undoubtedly produced the most interesting results of this experiment. The "orthogonal effect" has appeared in this task as well as the two previous ones, indicating once again the generality of this phenomenon. It is worth expanding on the type of errors children performed in the diagonal condition of the task as they give an interesting insight in children's problems with this task. When the table is presented in the orthogonal position with the horizontal base and the tilted table top, children very rapidly realise that they have to equalize the length of the legs in order to obtain a "flat" top which will support the ball. In fact they quickly move the legs up and down, obtain the horizontality of the plane and then put the ball on the top and assess whether it stays in equilibrium.

When the table is presented on the wedge with the legs of equal length and the tilted table top, nursery children seem puzzled as what to do. They know and express the fact that the ball will roll off in this state of the table. However their attempts to put the

table top "flat or level" are slow and uncertain. The same subjects that easily set the table top horizontally when the base was horizontal are at loss in the situation in which the table is on the wedge. These subjects that fail the diagonal condition, make various attempts in lengthening the legs or shortening them and every time they test their settings by placing the ball on the top and observing that it rolls off. A 4 year old after various attempts gave up saying " this ball is really very very rolly". These children start by lengthening the leg which corresponds to the lower side of the wedge. In fact they "work" only on this leg and tend to leave the left leg corresponding to the highest side of the wedge untouched.

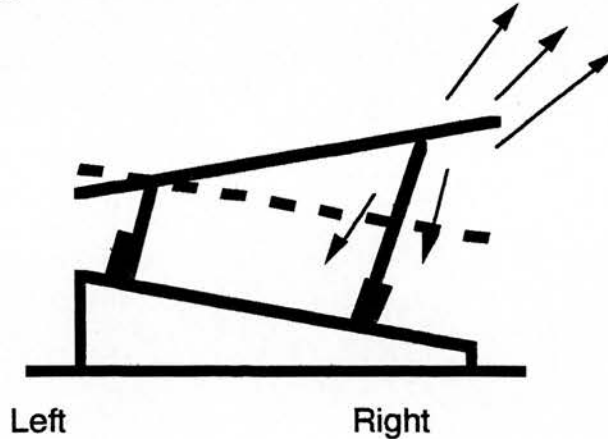


Figure 7.17 The right leg of the table which nursery children lengthen and shorten to obtain the horizontal table top.

These children realise that it is the right side that must be lengthened, however they never seem to lengthen it sufficiently. All the unsuccessful subjects set the table top tilted towards the lower side of the wedge, the ball always rolls off on the lower side of the wedge demonstrating that the child has not lengthened the corresponding leg sufficiently.

The problem seems to be the conflict between trying to keep the legs at the same length and having the table top horizontal. In fact often children started by lengthening the right leg to compensate the inclination of the wedge, then they had a look at the global configuration and shortened the right leg again in order to have the same length in the two legs. One child was very explicit in this attitude: I had introduced a small cylinder in one of the two hollow rectangles so that children in the orthogonal condition would not simply push the legs down to the bottom but had to actually move them up and down to obtain the same length. At a certain point during the interview of a little 4 years old girl, she was having great difficulty in setting the legs exactly at the same length, when the table was on the wedge. Out of curiosity, I

took the cylinder out of the hollow support. The child immediately pushed down both legs so that their extremities touched the base and the legs were equal length and said: "ahh that's much better". She then did not modify the length of the legs even after observing that in this condition the ball did not stay still but rolled off. In fact she attempted once more to lengthen the right leg but then pushed it down to the base and left it in this position.

Another nursery child was lengthening the right leg and was obtaining a horizontal top then leaned forward at the height of the legs and observed that they were different so he lowered the legs again and left them at the same length. When he noticed that the ball rolled off he still lowered the right leg which was the side on which the ball had rolled down. Finally another child who had succeeded in placing the table top horizontally, observing that the ball stayed in equilibrium said: "the ball stays still but it is squint!".

On the contrary children of Primary 1 and Primary 2 who succeeded in this diagonal condition were very rapid in their setting and behaved with the same assurance as the younger children do in the orthogonal condition. Primary 2 children in fact found the task rather trivial and put the table top horizontally at the first attempt.

In conclusion, the table task highlights once again the reliance of young children on the features of the object as opposed to an external frame. The subjects that fail the diagonal condition seem to expect that the table top stay "level" only when the canonical configuration of the table is maintained, when in other words the legs are kept equal and the top is perpendicular to the legs. Basically therefore the same arguments that have been introduced previously to explain the *décalage* between orthogonal and diagonal conditions of the other tasks, apply in this situation. The orthogonal condition does not introduce any conflict between the internal and the external relations. When the table is on the flat base the table top is at the same time perpendicular to gravity and perpendicular to the legs which are also symmetrical. The condition on the wedge instead, requires subjects to relate the table top to the external frame and to break the symmetry and perpendicularity of the figure.

7.6.5 The general hierarchy: the acquisition orders between the table and the other tasks

The overall order of acquisition of the three tasks that has emerged from this study is the following: practically all the subjects solve correctly and concurrently the orthogonal conditions of the balance and table, there is a *décalage* between these tasks and the orthogonal bottle. Then the diagonal table is acquired prior to the orthogonal

bottle. Finally the solution to the orthogonal bottle is followed, as in previous experiments, by the solution to the diagonal condition of the balance and the bottle.

The first interesting data to retain are the concurrence of the table and balance orthogonal and the *décalage* between these conditions and the orthogonal bottle. As I discussed above this *décalage* may be due essentially to the means of response and the difficulty children have in drawing straight lines. Within the age group examined therefore, when the task is practical or when the representation of the plane (the paper sticker) is given, children are capable of orienting a surface horizontally in the orthogonal conditions.

The second important result is the *décalage* between the correct solution to the diagonal table and the diagonal conditions of the balance and the bottle tasks, and with respect to the orthogonal condition of the bottle. The fact that the diagonal condition of the table is solved earlier than the other diagonal tasks dismisses the hypothesis that there is a general synchronous transition from an object centered to a framework centered competence in orienting horizontal planes. We cannot characterise children's representations of horizontality as purely object centered at one stage and purely framework centered at the next. However this conclusion does not really come as a surprise. In fact, in previous chapters I claimed that children are able to attribute internal axes to objects relying on the object's relation to the direction of gravity when in equilibrium. This claim which goes against Piaget's hypothesis that children are unable to refer the orientation of objects to the vertical/horizontal system of coordinates, is tantamount to saying that children have the capacity to make use of an external spatial framework. The evidence therefore, that young children are capable of practically orienting the table top in a condition in which the axes of the table do not correspond to the axes of the environment, confirms that by the age of 4 children are capable of referring an object to the horizontal and vertical axes of the environment.

There are two results which remain to be explained: the *décalage* between the orthogonal and diagonal in the table task, and the *décalage* between the diagonal table and the other diagonal tasks. In fact, if the child exhibits the capacity to refer the table top to an external referent why does he fail with the other diagonal conditions? Certainly the fact that the table task is a practical problem which does not involve the translation of the orientation of a plane onto the different "space of the page" may be playing a role, however the evidence that even the table task shows the presence of the "orthogonal effect" may suggest that there are other factors intervening in this developmental process. The fact that in this practical situation as well there is a developmental process that leads children from succeeding in the orthogonal condition only, to succeeding also in the diagonal condition indicates that referring the plane to

the external axes is not immediately available to young children and that even in this situation they must come to realise that the plane is oriented independently from the rest of the table. The delay between solving the orthogonal and diagonal conditions of the table task, shows that initially the child is relying on the correspondence between the axes of the table given by the vertical position of the legs, the horizontal base and the symmetry of the display. In the diagonal condition new axes have to be assigned to the display which requires that a different set of axes, those of the environment, be considered pertinent.

All the tasks have shown that assigning the appropriate axes to the display is made more complex when the external axes do not correspond to the axes of the object. When the child has to assign new axes to the configuration, be it the table or the balance, an additional complexity is introduced. Moreover, in all the tasks a conflict arises between the internal axes and external axes. Conflict that is resolved when children realise that they have to abandon the reference provided by the internal axes and that the orientation of the plane must be determined only by relying on the reference provided by the new set of external axes.

With regards to the *décalage* in the solution of the various diagonal conditions, the hypothesis can be made that the assignment of new axes and the resolution of the conflict between the two sets of reference axes, is of variable difficulty and is associated with the understanding of the physical properties of the object. Olson & Bialystock suggest that the assignment of new axes and the difficulty with which this is achieved depends on a variety of factors.

....primary axes are assigned to objects in the environment automatically, and to the extent that these axes are also the appropriate ones for the solution of a spatial task, the problem is easy to solve. Where a new set of axes is required to for the purpose of solving the task, the problem is more difficult. When the axes need to be reassigned to solve the problem, the basis of the reassignment can vary, and that difference accounts for some of the differences in task complexity. Moreover the greater the conflict between the two set of axes, the more difficult the problem. (1983 p. 238)

Physical knowledge, may hence be one of the determinants of the complexity of the tasks inasmuch the comprehension of the factors that are involved in the state of equilibrium of the plane underlies the research of the correct set of reference axes. It may be suggested that there is a progressive extension of the objects and planes that are recognised as finding their own equilibrium i.e. that have dynamic properties that make them independent from the position of their supporting elements. These objects must therefore be oriented with respect to the direction of gravity which determines their position and their spatial relations with the other parts of the objects to which they

belong. For each object employed in the experiments children come to discover the role of gravity and the consequences this has on the position of equilibrium the plane assumes.

Chapter 8: Conclusions

8.1 Introduction

At the end of the literature review on the water-level task (section 4.4), I raised the question of the nature of children's representations of horizontal and vertical planes in the stages preceding the correct resolution of the water-level problem. It was in fact suggested that neither Piaget's nor any of the following studies reviewed, could provide a specific account of children's understanding of the coordinate reference system during the long period leading up to success in the water-line task. It was argued that such a specification was necessary to capture the coherence behind children's behaviours in that period of development. There seemed in fact to be some apparently contradictory evidence as to the extent of children's competence or lack of it, as children could fail some conditions while succeeding other versions of the horizontality task.

The three previous experiments have attempted to answer this question by providing new data about children's competence in tasks of orientation and the conditions in which this competence is revealed in the stages preceding success in the classical water-level task. The results have however, done more than provide additional evidence on the "intermediate" stages we had set out to explore. They have challenged the view that there are two well distinct stages, one in which children do not possess a framework of spatial reference, and one in which children have acquired a coordinate system of reference. Two clear findings emerged from these studies, the "orthogonal effect" and the *décalage* in the solution of the three tasks of orientation (the balance, the water-level and the table). Across the three tasks, we have each time witnessed the same pattern of collective *décalage* between orthogonal and diagonal conditions. Recursively in every one of the situations examined, children are able first to orient a plane horizontally when it is perpendicular to the internal axes of the object and only later to represent the correct horizontal orientation when the plane is not aligned with the axes of the object it is supported by. This new competence however, appears at different moments for each task. Children can correctly orient and represent a horizontal plane both for the orthogonal and diagonal orientations of the supporting object, at different moments of development depending on the particular context. In other words, there seems to be no "once and for all" transition from a stage in which children's spatial representations are not organised around a coordinate system of

reference to a stage in which all orientations are determined on the basis of the vertical/horizontal axes of the environment.

In the next sections I will argue that these results cannot be explained by any of the approaches that were presented in Chapter 4, but neither can they be interpreted as a general transformation from an "object centered" to a "framework centered" stage as I had initially proposed. On the contrary the results indicate that subjects can rely on one or the other spatial systems depending on their understanding of the physical properties of the object they face. Children's difficulties in orientating the planes and lines are borne from the conflict between two reference systems (the internal axes of the object and external axes of the environment), and from the choice of the salient referent. The correct choice of reference axes depends on a variety of factors of which the most significant seems to be a correct understanding of the dynamic properties which determine the relative independence of the plane from the other elements of the object.

8.2 A summary of the results in the three experiments

Let us first of all recall briefly the most significant results of the three experiments.

1) In the three tasks used, the table, the balance, the water-level, there is a clear developmental *décalage* between the orthogonal and diagonal conditions. Subjects that are able to orient the plane horizontally in the diagonal condition necessarily are capable of doing it in the orthogonal condition, while there are many subjects that fail the diagonal condition while succeeding the orthogonal. This is what I have called the orthogonal effect. The two conditions differ in the spatial relation holding between the object and the environment and in the spatial relation holding between the horizontal plane and the other elements of the object supporting it. In the orthogonal condition the internal perpendicular axes of the object be it the table or a bottle coincide with the vertical and horizontal axes of the environment, in the diagonal condition no such correspondence exists.

Figure 8.1 Stage sequence in the solution of the series of horizontality tasks

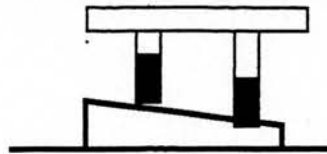
Level 1

Orthogonal positions in:



Level 2

Diagonal position in:



Level 1b

Orthogonal positions in:



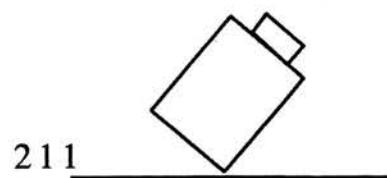
Level 3

Diagonal position in:



Level 4

Diagonal position in:



2) There is a *décalage* in the solution of the diagonal condition across the tasks. The table diagonal is the first to be solved around the age of 5. This ability precedes the capacity to depict the water-level horizontally in the bottle. It is followed by success in the balance and round container for which there is a weak effect of concurrence. These conditions are succeeded around the age of 8. There is therefore a rather long delay from succeeding the orthogonal conditions of these tasks to succeeding the diagonal conditions as well. Finally much later, the water-level in the square container is correctly oriented in all its conditions. The delay between succeeding the orthogonal and the diagonal conditions in this task, is very long indeed. Many of our oldest subjects failed the diagonal condition of the square bottle a finding compatible with the report of a significant number of adults failing the task.

Overall, what appears from this summary is a rather fragmented picture in which there is little homogeneity in the acquisition of the various tasks and task conditions. The figure below summarises graphically the sequence of task conditions successfully solved as it has emerged from the orders identified in the experiments. I distinguish 4 levels corresponding to the orders emerged from the three experiments.

8.3 The import of the results on Piaget's theory of the development of spatial coordinates.

According to Piaget & Inhelder before being capable of correctly representing the orientation of the water-level in all the positions of the bottle, the child cannot be attributed with the capacity of relating the position of an object or plane with respect to the system of natural coordinates, horizontal and vertical. In the stages which precede this competence the child is either only considering topological properties of spatial relations or is in the process of constructing the spatial system that will allow him to relate all objects and virtual positions to a unique fixed system of reference. As was discussed in Chapter 4, for the authors the failure of the young child to organise spatial relations around a system of coordinate axes, is due both to the lack of operativity of his representations as to the lack of the conceptual apparatus of euclidian geometry.

Two types of results have emerged from our studies which challenge Piaget & Inhelder's interpretation at two different levels: the sophisticated use of euclidian properties in the correct solution of the orthogonal conditions of the tasks, and the *décalage* in the capacity to refer a plane to an external frame of reference in the diagonal condition across the various tasks. Taken together, the findings of our three experiments indicate that the clear-cut distinction between subjects that represent and

employ a system of coordinates and subjects who do not, is arbitrary, as different contexts elicit different performances at different ages.

First let us consider the import of the finding of an "orthogonal effect" on Piaget's explanation. In experiment 1, it appeared that the water-level was represented horizontally concurrently for all the orthogonal orientations of the bottle and that this ability preceded the correct resolution of the task for the diagonal tilts of the bottle. Whereas Piaget & Inhelder described the initial stage as one in which correct performance was found only in the upright canonical position of the bottle (this phase was followed by the capacity to represent progressively the horizontal water-level for the other orthogonal positions), it was found that as soon as subjects start depicting the water-level with a line they represent the water horizontally for *all four* orthogonal positions of the container. Contrary to the view that, at a first stage, subjects are unable to reproduce correctly none but the canonical position of the bottle, this evidence suggests that there are a number of orientations that are successfully represented. The result is less surprising if one considers that the category of orthogonal orientations of the bottle that are found easier, share a certain number of features as do the "difficult" diagonal orientations. The orthogonal conditions all share the perpendicularity of the water-level with the sides of the bottle, the parallelism of the line with one of the sides of the container and the coincidence of the vertical and horizontal axes of the bottle with the axes of the environment. The diagonal conditions all share an angle of incidence between water-line and sides different from 90° , a lack of parallelism of the line with any of the sides of the container and a mismatch between the axes of the bottle and the axes of the environment.

From our investigation with the spherical container it resulted that it is the coincidence of internal and external axes or the lack of it, that makes a difference for child and helps to generate the correct or incorrect response. In fact, in the sphere, in absence of straight sides, the orthogonal condition differs from the diagonal only inasmuch its main axes coincide with the external axes (the relation between the water-line and the sides remaining invariant). The finding of the orthogonal effect with the round bottle suggests therefore that it is on the basis of the correspondence between internal and external axes, that the orthogonal conditions are correctly solved before the diagonal tilts.

The finding that children are initially capable of orienting the water-level horizontally in all the orthogonal conditions of the container suggests a number of remarks as to their competence in representing the horizontality of planes. The first consideration regards the spatial conceptions of those "intermediate" subjects described by Piaget & Inhelder. When subjects succeed in the orthogonal conditions, despite their failure in

the diagonal conditions, they are employing relations of perpendicularity and parallelism which are relations of euclidian geometry. By drawing the water-level horizontal when the bottle is upright or at a 90° angle of inclination, they are establishing a relation of parallelism between the line and the horizontal axis of the bottle and a relation of perpendicularity with the sides and the vertical axis of the bottle. Furthermore we have seen how in absence of straight sides children can accurately refer the water-line to the "imaginary" axes of the sphere. These axes are attributed on the basis of position of the spout and form a spatial framework internal to the sphere. This attribution as well, seems to imply sophisticated knowledge of euclidian relations. It seems to be the case therefore that in the stages preceding the correct solution of all the conditions of the water-level task, subjects are perfectly capable of representing and using euclidian spatial relations.

The Piagetian view that in these preliminary stages, children are relying solely on topological relations seems to be inadequate. Even if we were not to consider the evidence of early success in the table and balance tasks, failures in the diagonal conditions of the water-level task cannot be attributed to the lack of euclidian spatial relations, given their concurrent behaviours in the orthogonal conditions. Furthermore the evidence of the concomitant success in all the orthogonal conditions indicates that the difficulty these subjects are facing is of a different nature. The facilitatory effect of the orthogonal conditions seems to be caused by the coincidence of the internal and external axes that relieves subjects from having to refer the water-line to the more general framework given by the direction of gravity. Subjects can limit themselves to the relations holding within the bottle (between the water-level and the sides) and ignore the relation of the water-line with the environment. Since the internal axes of the bottle in the orthogonal conditions, coincide with the external horizontal and vertical axes, they can be successfully taken as reference. The child, therefore, operates adequately within the space of the bottle which provides an object centered framework, coinciding with the larger framework of the environment, to which the water is related.

In summary, the orthogonal effect seems to challenge Piaget's and Inhelder's claim that young children lack the apparatus of euclidian geometry. As for the second level of Piaget & Inhelder's explanation, the limited representational mobility of the preoperational child which prevents young children to take into account a distant referent such as the horizontal, the data on the *décalage* in the solution of the diagonal condition across the different tasks raises a few questions as to the role this factor is playing in the overall development of the notion of horizontality.

The *décalage* in the acquisition of the diagonal conditions of the three tasks is difficult to explain from the perspective of a global change in the representation of space between the pre-operational and concrete operational stages. The diagonal conditions of the tasks are those, we have argued, that require subjects (more so than the orthogonal conditions) to orient the plane using the axes of the environment as reference. If, in the orthogonal condition the coincidence of internal and external axes was simplifying the task, in the diagonal condition the lack of such coincidence demands that the child consider the horizontality of the plane solely with respect to the direction of gravity and its perpendiculars. This ability corresponds to the competence that Piaget & Inhelder attributed to an operational spatial structure where all planes are oriented with respect to the coordinate axes. Therefore, the fact that the diagonal condition is solved correctly at different moments for different tasks implies that at different moments in development children exhibit the competence that these authors attributed only at the end of the construction of the spatial reference system.

On the other hand, the very early solution to the table task could be seen as depending on the practical nature of the task. From a Piagetian perspective, the table task may be considered a task which involves processes of a different level (orienting a real plane) from those involved in the water-level task in which the water-line must be depicted (orienting a graphic representation). While partially agreeing with this point, I would want to underline the fact that the task is done in anticipation: children set the table-top in a certain position and then test the equilibrium of the ball. It is not a problem in which children can simply adjust the plane by trial and error strategies. I would rather argue that the capacity to orient the table top horizontally in the diagonal condition does indeed demonstrate, an ability to envisage the independence of that plane from the other elements of the object and to refer it to a distant feature of the environment. One of Piaget's arguments to explain children's failure in the water-level task was that children were fixed on proximal relations and were unable of considering the spatial relations holding between the plane and distant objects. Success in the table task shows that already at 4 or 5 years of age and well before success in the water-level task, children demonstrate the ability and representational mobility to relate a plane to the direction of gravity and of abandoning the attention on the sole relations internal to the object. Should it even be the case that the task is only involving procedural knowledge as opposed to representational knowledge, on which I would have some serious doubts, the fact remains that the young child can take into account non-proximal spatial relations to anticipate the correct orientation of the table top.

The results with the round container and the balance, also, introduce evidence of early competence on horizontality that may be considered less controversial than the table

situation, as both tasks require a symbolic representation to be oriented. Both the round container and the balance elicit correct responses in the diagonal condition before the square container. I do not wish to address once again the issue of early acquisition and the import of such data on Piaget's theory (it has been amply discussed in Chapter 1). However the *décalage* between the various tasks casts some doubts on the univocal transition from a stage in which children are unable to refer a plane to external axes to one in which this competence is acquired. For the latter explanation to be adequate we would expect that the capacity to take into account the existence of an external framework to which the plane is to be referred, would generalise to different conditions. Once the child has demonstrated the capacity to represent such a framework and the ability to use it as reference for the orientation of an object, it is difficult to explain why he would revert to an object centered reference in new situations. Thus the problem we face, is not so much that of identifying the moment in which the child's representational system has developed to the point of having the necessary operativity to represent the abstract system of coordinates, but that of explaining why he employs this knowledge in certain contexts and not in others. We are, in other words, confronted with the question of what are the reasons that allow children to orient a plane horizontally by referring the plane to an external feature of the environment before another plane, given that subjects are capable of such a relation in the context of the table and then of the balance. Piaget's explanation cannot provide any satisfactory answer to this question.

In conclusion it appears that there are no grounds to consider that children's errors in the water-level task are due to a system of spatial representation that could be characterised as non euclidian. Neither does it seem the case that it is by lack of representational mobility that children do not consider the relation between a plane and an external framework. In each of the tasks studied the orthogonal conditions are solved first by employing some euclidian relations (although internal to the object), the diagonal conditions are solved by employing these relations to determine the relation holding between the plane and features of the environment.

8.4 The import of the results on Ibbotson & Bryant's explanation of the errors in the water-level task.

I would like to briefly reexamine the interpretation of the water-level task proposed by Ibbotson & Bryant at the light of our results. It may be argued in fact, that the orthogonal effect I have identified is an homologue of the perpendicular effect highlighted by these authors. Recall that Ibbotson & Bryant reported that children not

only found the reproduction of perpendicular angles easier than non perpendicular angles but also showed a tendency to perpendicularise the non perpendicular configuration, i.e. to draw or reproduce the angle more towards the perpendicular than the opposite. Ibbotson & Bryant suggested that this perpendicular bias in children's reproductions of angles also explained their mistakes in the water-level task. The fact that in my experiments children find the orthogonal conditions (in which the horizontal plane is perpendicular to the supporting elements) easier than the diagonal conditions (in which the angle formed by the plane and the other elements of the objects are greater or inferior to 90 degrees) may go in the direction of this explanation. However two sets of results cannot be explained by this interpretation: the orthogonal effect for the round container and the *décalage* in the solution of the various diagonal conditions. The fact that we have found a similar facilitatory effect of the orthogonal positions of the round bottle in which there is no straight baseline from which the line representing the water-level departs, and therefore no perpendicular angle between the side and the line to be oriented, invites to a certain caution in considering the perpendicular error the only source of failure in this type of task. I argued that the sphere is *attributed* internal axes to which the line is referred, but this seems to be a different effect from that described by the authors for situations in which a baseline was always present rather than imagined. The clear developmental order that holds between orthogonal and diagonal orientations of the round bottle cannot be attributed to the contrast between a line that is drawn perpendicular to its baseline or at an oblique angle with the baseline as is the case in the square bottle for instance. In all of its positions, the sphere maintains the same angle of incidence with the water-line.

Furthermore although Ibbotson & Bryant do not make any claim as to the developmental process that would lead the child to eventually abandon such a perpendicular bias, it is plausible that the authors consider that when tasks such as the water-level are correctly solved, children have overcome the perpendicular drawing error. On the contrary, we have seen that there is a *décalage* in the solution of the diagonal conditions of the different tasks and that there are cases of 12 year old subjects still failing the diagonal condition of the square bottle. The bias would therefore seem to persist until a late age in some conditions and disappear in others. Without wishing to question their results I would simply suggest that the perpendicular bias is not sufficient in itself to explain all the behaviours we have observed although it may certainly be an associated factor.

There is however one point on which Ibbotson & Bryant's argument is compatible with the evidence collected in previous experiments and this is their report of a vertical effect. When the baseline is vertical the perpendicular error was reported to disappear.

The authors admit of being rather puzzled by this finding and attempt to explain it in terms of the coincidence between the internal sense of postural verticality and the external framework of the environment. Ibbotson & Bryant suggest that such correspondence leads the child to start organising his spatial representations around the vertical axes and to consider the vertical the main reference point in space, which thus enables him to solve some spatial problems before others. Here too an hypothesis of the primacy of vertical orientations in children's spatial knowledge similar to the hypothesis I am suggesting, is invoked to explain the facilitatory effect of contexts in which there is a coincidence between the direction of gravity and the internal axes of the objects.

8.5 A reinterpretation of the development of horizontality

The objective that had been set at the end of the literature review was to capture the coherence behind children's answers at the water-level task in Stages IIa, IIb, IIIa of Piaget's classification. In other words, I wished to characterise the spatial conceptions of the long period preceding success in all the conditions of the task, conceptions that I argued, could not be described in terms of a topological spatial system as Piaget claimed, nor could they be described as fully euclidian, given the failure in the task. I was therefore, looking for additional evidence that would allow us to provide a *specific* characterisation of the spatial conceptions of this "intermediate" period. The underlying hypothesis behind this quest, was that children had a level of spatial organisation that possessed some of the features of euclidian geometry (as was evident from the correct responses to some of the bottle orientations) but lacked other features of an adult geometry.

This objective and this hypothesis underlie the conclusions to Experiment 1 in which an "object centered geometry" was proposed to capture the spatial conceptions of subjects who could represent the horizontality of the water-level only if the container was orthogonal. The concurrence of the square and round orthogonal and their décalage with the diagonal conditions, seemed to invite the hypothesis that there is a stage in which subjects are object centered and a following stage in which subjects are framework centered.

I borrowed this distinction from Olson & Bialystock, as it seemed to discriminate subjects on the basis of the referent chosen rather than the type of relations employed; a viable solution for characterising a spatial system that employed euclidian relations such as perpendicularity etc., but only for relations holding within the object. The

difference between an object centered geometry and a framework centered geometry as defined by Olson & Bialystock, seemed to me to be principally in the type of referents chosen rather than in the type of relations. Therefore rather than opposing as Piaget, a topological system in which the type of relations envisaged are radically different from those of the euclidian system (inclusion, proximity etc. versus parallelism, angles, distances), the stages of development of horizontality could be distinguished solely on the base of the choice of reference system. This solution seemed to be compatible with the "orthogonal effect" and with the finding that children could structure the object in a complex system of vertical and horizontal axes while yet failing to represent the water-level horizontally when the position of the bottle created a mismatch between internal and external referents.

Furthermore, the findings of Experiment 2 which showed concurrence between all the orthogonal conditions including the new balance task, went in the direction of a general object centered stage. It appeared therefore that the characterisation of the "intermediate" stage of Piaget's & Inhelder's description could be carried out invoking the "object centered" geometry. Not only had the right type of description been found to characterise the underlying coherence of the patterns of errors and successes in children's responses, but a stable stage organised according to this spatial system could be defined.

In Experiment 2, however, it also appeared that there is a *décalage* in the solution to the diagonal conditions of the various tasks. These *décalages* were confirmed in Experiment 3 in which the table task was solved much earlier than the other tasks. Thus, the argument that we brought against Piaget in section 8.3, namely that there are no grounds to consider that there is a radical transformation in the representational capacities of subjects failing and succeeding the water-level task, also applies to the hypothesis of an object-centered to framework-centered shift. Evidence does not support the view that there is a global transformation in children's spatial representations of the horizontal.

If both the framework related and object related systems are available what brings the child to rely on one or the other? In which contexts children make use of an "object centered" geometry and in which do they use a framework centered system? There are in fact many cases in which internal axes are an adequate reference for the horizontal orientation of the plane.

In many cases solid objects behave as wholes, their dynamic properties involve the object in its totality. If we take a solid like a chair, the rotation of the chair maintains the relation between the parts of the chair unchanged. The chair rotates with regards to the environment and therefore all of its parts change of position with respect to an

external framework. At the same time, they are not transformed in their relation to each other. Having identified the relation of perpendicularity that holds for instance, between the back of the chair and the seat, this remains unchanged across any possible rotation. Once the major axes of the chair are defined and determine the mutual relation of the various parts, the relations remain invariant and can be repeated or reconstructed by relying on the canonical position of the chair. In these cases it is correct to use the axes of the object to determine the mutual relation of the various parts of the object when it changes position. An object centered geometry is therefore adequate to represent the relations *between* parts of objects. It delimitates space to the relations within the object.

Object centered geometry is functional to describe objects and to represent them as a set of elements that are spatially related between each other. The same pieces of wood that make up the chair put in a different spatial relation to each other give rise to a different object, a stool for instance. Furthermore, these object centered relations reflect some of the features that determine the state of equilibrium of objects. The fact that our manufactured world is made of objects which are symmetrical and which are constructed as to have perpendicular angles between its parts (as the back and seat of the chair) is due to the fact that these relations between the parts of the object guarantee the coincidence of their axes with the direction to gravity and the perpendicular to it. In order for an object to be a stable support it must have a plane that is horizontal to counteract the pull of gravity. The horizontality of this plane itself is achieved by its perpendicularity to the vertical elements of the object, which are themselves made as to assume a position coinciding with the vertical of the environment. In many cases therefore, in the position of equilibrium of the object the major internal axes correspond to the external horizontal and vertical.

What is essential however, is that in order to identify the axes of the object, and to determine its position of equilibrium, children cannot but refer to the horizontal and vertical of the environment. The identification of the canonical position of the object passes through relating it to an external framework. We could say that in order to create the specific object space the child has to pass through a framework centered description. The axes of this framework are in fact organised around the direction of gravity which is the principle feature of the environment and from which the horizontal is derived. It is with respect to these features that objects are given their canonical spatial descriptions inasmuch identifying a canonical top and bottom and a major vertical axis, requires relating the elements of the object to an external frame. As far as the child is concerned therefore, the horizontal axes of the object embeds the horizontal

axis of the environment. The axes of the object are a realisation within the object of the axes of the environment.

The problem arises when the two set of axes, internal and external, do not correspond any more, and a part of the object has to be reassigned an orientation independently from the axes of the object. In this situation there are two sets of axes that have to be considered and which are in conflict. The object has its canonical axes which defines it as an object with specific spatial characteristics, or as Olson & Bialystock claim:

...constitute part of the implicit structural descriptions which underlie the perception and representation of objects and events. (1983 p.54)

and are permanent and invariant properties of the object. Simultaneously the child must consider the second set of axes given by the direction of gravity and its perpendicular. The child has to work out that it is in fact to this set of axes that the plane must be related. The developmental problem is hence that of deciding which set of axes is pertinent to the situation and the most accurate reference. To quote Olson & Bialystock again:

Water level may be tilted relative to the the axis of the bottle, which may be tilted relative to the axis of the table, which may be tilted relative to the room, which may be tilted relative to the house, hillside, the horizon, and so on. Which axes are more salient and which are appropriate for various judgments is therefore a complex issue. (1983 p. 236)

In sum, it appears that the developmental progress corresponds to the ability to make the correct choice of reference axes and to resolve the conflict between the different axes assigned to the display. At a first level the child has a coherent theory according to which the plane is in equilibrium when it lies on the horizontal axes of the object, it is thus perpendicular to gravity and stable. Maintaining the relation of perpendicularity with the vertical axis of the object guarantees for the child the correct representation of horizontality and stability. From the perspective of this theory, when there is a mismatch between the internal and external axes a plane which is horizontal with respect to the environment but not with respect to the internal axes, is considered tilted or squint, as scottish children say. At the second level the child comes to consider that the plane is stable and in equilibrium even when it is not parallel to the horizontal axes of the object. The conflict between internal and external axes is resolved in favour of the external set of axes. From the perspective of this new theory, the plane is horizontal or "level" regardless its relative orientation to the internal axes. Now it is the object that is "squint".

But how do children come to reassign the axes and to choose the correct and more salient set? I would argue that it is the realisation of the dynamic properties of the object and in particular the plane, under the effect of gravity, which determines eventually the correct choice. However, addressing the question of whether there is an evolution in the range of physical situations for which the child relies automatically on a framework centered system and for which the resolution of the conflict between internal and external axes is done more easily, would require more evidence on the development of children's understanding of problems of dynamics and equilibrium.

8.6 Conclusions and future lines of research

The first three chapters of the dissertation had introduced a general framework with which to approach questions of conceptual development. This framework had as major assets the idea that there were no general changes in children's representational format but that developmental progress went on at the level of specific domains of knowledge. It was also suggested that conceptual development corresponded to the progressive extension of concepts to comprise new contexts. This view determined somewhat the experimental questions we set out to explore. In fact as was frequently pointed out, the objective was to capture the *transformation* of the concept in development rather than identifying when it emerged as a "full blown" concept. I claimed that the specification of the contexts and contents to which the concept was successfully applied at various moments of development, could help to define the extension of the concept at different periods of the child's evolution.

The results of the experiments proved to be compatible with this approach in more than one way. As far as the idea that there are no radical transformations in children's representational power, transformation that allows a certain concept to emerge and be entertained by the more mature and mobile representational system, the results have suggested that in the case of the horizontal the child has the representational capacity to entertain the concept from early on. The evidence of successful orientations of planes in some contexts dismissed the hypothesis that young children do not represent spatial orientations in terms of a horizontal and vertical system of reference. The evolution of children's conceptions does not occur at the level of the acquisition of a novel set of orthogonal axes that serve as referent for the orientation of objects and their parts, but rather in the capacity to choose the most relevant system of reference for the specific situation they face. The assignment of axes to a display which is a physical display in the world, comes to respect the dynamic and static properties of that particular display.

Hence the transformation of children's competence in orienting objects goes hand in hand with the transformation of their notions of equilibrium.

In the theoretical chapters I also presented the view that it was possible to characterise the coherence behind children's conceptions at a given stage in the development of a concept, in terms of a theory organising their beliefs and behaviours. It was suggested that "theories" were a structured system of beliefs that underwent radical transformations from a stage to the next. It was claimed that if there was little evidence for a major restructuring in the representational system of the child at different ages, developmental change could be explained as local restructuring. Theories or local structures underwent a process of extension and specification in development which corresponded to a radical restructuring and redefinition of their elements. The discontinuity of conceptual development could therefore be captured by the idea that at each stage of the development of a concept, there was a coherent system definable as a theory which gave rise in the following stage to a more extended and more powerful theory. Children's capacity to solve increasingly a greater number of problems mirrored the progressive extension of their implicit theories.

The initial objective of the dissertation was to provide this type of account for the development of the horizontal and vertical coordinate system. The results of the studies have however indicated that the increasing ability to define their orientation in space goes hand in hand with a change in children's theories of dynamics and equilibrium. What the data indicates, is that there is a progressive extension of the objects for which the child is capable of choosing the most salient reference system. There is in fact a progress in the child's competence to identify the perpendicularity of certain planes with the direction of gravity. With age children acquire the increasing capacity to evaluate the dynamic properties of objects and the state of equilibrium they assume under the effect of gravitational force. The analysis of the transformation of children's theories of orientation can thus, only be achieved by a joint analysis of the development of their theories of equilibrium. The data collected in the previous experiments cannot provide the basis for a detailed account of the transformation of the physical notions, but simply indicates a direction of enquiry. A very tentative suggestion, may be, that the child has an initial theory according to which, the position of equilibrium of a plane supported by solids is such, that it corresponds to the property of being on the horizontal axis of the object. Later this theory is modified into a theory according to which the plane is in equilibrium when it is perpendicular to gravity.

In an interesting study in the domain of adult's naive physics, Roncato & Rumiati (1985) studied adult's understanding of phenomena of equilibrium. They presented

subjects with a rod supported by two bars of unequal length and by a rope attached to its middle point. The rod sitting on the unequal bars was tilted to one side. They asked subjects to predict how the rod would orient itself when the two bars were taken away and it remained supported solely by the rope. All the subjects predicted wrongly that the rod would balance until it found the horizontal position and there it would be in equilibrium. In fact the rod hung in its centerpoint remains tilted because the center of gravity, which corresponds to the centerpoint on which the rope is tied, is at its lowest point even when the rod is tilted on one side. Subjects argued that the rod was symmetrical and the two sides on the right and left of the centerpoint were therefore of equal weight, hence the rod would set itself "level".

What this study shows is that even in an adult population horizontality and equilibrium are strongly associated. Replicating this study with children, could constitute an initial direction of enquiry. In conclusion, in order to provide an adequate characterisation of the child's representation of the orientation of physical objects further evidence must be collected on the integration of physical and spatial knowledge.

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